# Copulas Cannot Solve the Simultaneity Problem*

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July 15, 2026

**Abstract**

We study the conditions under which the Gaussian copula correction of Park and Gupta (2012)—a widely used instrument-free method for demand estimation, with over 600 published applications across marketing, strategy, and adjacent fields—identifies structural parameters. We show that the correction is consistent only when three conditions hold jointly: a single unobservable drives the endogenous variable, its marginal distribution is known, and the mapping from the unobservable to the endogenous variable is strictly non-linear. The single-unobservable condition implies that the method can be consistent only where no valid instrument exists. That is, rather than dispensing with the exclusion restriction, the researcher must defend a stronger, untestable assumption: that no excluded variable shifts the endogenous variable. We then prove that in settings when the endogenous variable is determined in equilibrium by a system with multiple unobservables—supply and demand, oligopoly competition, or optimal advertising—the structural parameters are not identified even from the population distribution of the data. In simulations of these environments, the copula correction is inconsistent, sometimes of the wrong sign and farther from the truth than OLS, and it fails exactly where an excluded instrument recovers the truth.

**Keywords:** Endogeneity; Gaussian copula; Identification; Demand estimation; Simultaneity

**JEL Classification:** C13, C26, D12, M31

*We thank Yi Qian and UBC seminar participants for helpful comments. All errors are our own. All code used in this paper can be found at https://github.com/tomdavidoff/copula

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# 1 Introduction

Estimating market demand is a core part of the research program in both industrial organization and marketing. The central obstacle is endogeneity: prices are chosen by firms that observe demand conditions that the researcher does not, which induces correlation between prices and the structural error of the demand equation. The classic solution is to find demand or supply shifters that can serve as instruments (Wright, 1928). Valid instruments are often unavailable, however, and this has motivated a literature on instrument-free methods, including identification based on higher moments (Lewbel, 1997), heteroscedasticity (Rigobon, 2003), and latent instruments (Ebbes et al., 2005).

Within this literature, the Gaussian copula correction proposed by Park and Gupta (2012) has become a widely used instrument-free method. Qian et al. (2026) document over 600 published applications across marketing, strategy, information systems, and adjacent fields, with applications spanning pricing, promotion, and sales-force decisions. The method models the joint distribution of the endogenous regressor and the structural error using a Gaussian copula and adds a generated regressor—a non-linear transformation of the endogenous variable—to the structural equation. The method's appeal is evident: it requires no exclusion restrictions and is implemented with a single additional regressor.

Park and Gupta (2012) ground the copula correction in a statistical framing of the endogeneity problem: the researcher specifies a joint distribution for the endogenous regressor and the structural error, and identification follows from the parametric structure of that joint distribution. In this framing, the endogenous variable is a monotone transformation of a latent variable that lacks a clear interpretation within an economic model. When modeling economic environments, by contrast, the endogenous variables are determined by solving a system of equations—a market's equilibrium price and quantity are computed by equating the supply and demand curves of that market—and the endogenous variable is a function of all the exogenous variables and unobservables in the system. In these settings, endogeneity arises due to the simultaneity of multiple equations, each with its own source of unobserved variations (Haavelmo, 1943). In this paper, we frame the endogeneity problem around the economic structure of price determination and ask which data-generating processes for the endogenous variable are consistent with the assumptions that the copula correction requires. Framed this way, the method fails in the settings in which it is most often applied.

We make three contributions. First, we identify the conditions under which the copula correction delivers consistent estimates. We show (Theorem 1) that when the endogenous variable is generated by a single-equation process—a pricing equation driven by a single unobservable—the copula correction point-identifies the structural parameters provided that three conditions hold jointly: (i) the pricing equation is strictly monotone in a single unobservable, (ii) the researcher knows the marginal distribution of the endogenous component of the structural error—the component that enters the pricing equation, and (iii) the pricing equation is strictly non-linear, so that the endogenous variable and this error component are not identically distributed up to an affine transformation. Each of the three conditions is necessary: with a second unobservable entering the pricing equation, the correction term no longer recovers the endogenous error component; with an unknown marginal distribution, the correction term is a misspecified proxy and the residual remains correlated with the endogenous regressor; and with a linear pricing equation, the correction term is collinear with the endogenous regressor.

An excluded instrument is a variable that shifts price while remaining orthogonal to the structural error. The single-unobservable condition rules out any such variable: the correction can be consistent only where no valid instrument exists, and it fails wherever one does. Park and Gupta (2012) state: "Researchers routinely devote a great deal of effort toward convincing the reader that their assumed exclusion restriction is justifiable, but this claim is often debated (Conley et al., 2012). Note that this claim is impossible to test directly because of unobservability of the error." Our results imply that a researcher using the copula correction must convince the reader that no excluded variable shifts price while remaining orthogonal to the structural error—that no valid instrument exists. This claim is untestable for the same reason the exclusion restriction is, the unobservability of the error, and it is the stronger claim: the researcher must rule out every excluded shifter, not defend the validity of a single one. In setting out the econometric problem of simultaneity, Haavelmo (1943) emphasizes that, in real-world settings, each equation in a system is likely driven by its own unobservables.

Second, we prove non-identification when the endogenous variable is determined in equilibrium by a system of equations with multiple unobservables. In a supply–demand system, the equilibrium condition implies that the difference between the demand and supply shocks is a deterministic function of the equilibrium price. We show that the demand function is not point-identified from the joint distribution of prices and quantities: there is a continuum of well-behaved supply–demand structures that are observationally equivalent to the truth (Proposition 1). The copula correction—like any control function built from the endogenous variable alone—selects a single member of this continuum, generically not the true demand function. We further show that no regression of quantity on functions of price alone—the copula correction term, the raw CDF of price, or a polynomial in price—can distinguish demand from supply (Proposition 2).

The same logic extends to oligopoly games in which prices are determined by Bertrand–Nash equilibrium behavior: with $J$ products and $2J$ unobservables (demand shocks and cost shocks), the price coefficient is not point-identified absent classical exclusion restrictions (Proposition 3). The core issue is one of dimensionality: no function of $J$ prices can recover $k \times J$ unobservables with $k \geq 2$. To clarify what is missing in the instrument-free approach, we also characterize how an excluded instrument breaks the observational equivalence that underlies our non-identification results.

Third, we illustrate the performance of the copula correction in economic environments commonly studied by economics and business academics, using Monte Carlo simulations. In a supply–demand system, the copula estimates of the demand and supply slopes are both biased, and the copula method may even yield estimates that are more biased than OLS. In a monopoly pricing model with logit demand, the copula correction deviates from the truth when the single unobservable assumption is violated (i.e., demand shock and marginal cost variation), yielding estimates farther from the truth than OLS. Notably, marginal-cost variation is exactly the variation that makes a cost-shifting instrument feasible: in the same simulations, two-stage least squares with the cost shifter as an instrument is centered at the truth. In a duopoly with Bertrand–Nash pricing, each firm's price depends on both firms' demand shocks, and the bias of the copula correction grows with the variance of these shocks. We close with a model of advertising choice in which two underlying shocks—the demand shock and marginal cost—collapse into a single composite error. Estimation of advertising elasticities has been a common application of copula methods in recent years. We find the bias is as large as that of OLS and sometimes significantly larger.

Our research also contributes to a recent literature evaluating the copula correction. Eckert and Hohberger (2023) evaluate the finite-sample performance of the Gaussian copula approach and document sensitivity to departures from the method's distributional requirements. Qian et al. (2026) provide a practical guide that addresses implementation aspects of copula correction, including the construction of copula control functions and the handling of higher-order terms and non-continuous endogenous regressors. The authors also address situations in which the copula method may fail (e.g., multicollinearity of the endogenous variable and the copula correction term, misspecification). Hu et al. (2025) relax the Gaussian dependence structure with a non-parametric copula control function.

These contributions concern the conditions under which the method works as intended and how to implement it well. We contribute to this strand of the literature by focusing on a more fundamental identification issue: we show that when the endogenous variable is an equilibrium object determined by multiple structural shocks, the structural parameters are not identified even with knowledge of the population distribution of the data—that is, even with an infinite sample and a correctly estimated marginal distribution of the endogenous variable. The failure we document is one of identification, not implementation: it is not repaired by flexible copulas or by non-parametric estimates of the CDF.

Our paper is related more generally to the control-function literature. Imbens and Newey (2009) establish conditions under which the conditional CDF of an endogenous variable is a valid control function in triangular systems; their conditions require both a scalar disturbance in the reduced form and instruments that shift the endogenous variable while holding the control variable fixed. Instrument-free copula methods cannot satisfy these conditions in simultaneous systems: the reduced form for the equilibrium price contains both demand and cost shocks, and there is no exogenous variation available to trace out the structural function. Our Theorem 1 can be read as the special case in which the Imbens and Newey (2009) conditions hold without instruments because the reduced form is degenerate in all arguments other than the scalar disturbance; our non-identification results show that this special case excludes equilibrium price determination. The supply–demand version of our argument is, in essence, a restatement of the classic observational-equivalence problem (Working, 1927) in the language of the copula correction: the correction term is a function of the endogenous variable alone, and functions of the endogenous variable alone cannot separate the structural function from the conditional mean of the structural error. Relatedly, Rossi (2014) cautions against instrument-free approaches on the grounds that they substitute distributional assumptions for economic restrictions; our results give that caution a precise form in equilibrium settings.

The rest of the paper is organized as follows. Section 2 presents the canonical framework for the copula correction. Section 3 establishes the identification result for single-equation data-generating processes. Section 4 presents the non-identification results for simultaneous systems and characterizes how instruments restore identification. Section 5 extends the non-identification results to oligopoly games. Section 6 presents the simulation evidence. Section 7 discusses practical considerations, and Section 8 concludes.

# 2 The canonical framework

Park and Gupta (2012) consider a model such as

$$q_t = \alpha_0 + \alpha_1 p_t + \xi_t, \tag{1}$$

where $p_t$ is an endogenous variable in that it is correlated with the error term $\xi_t$. To address the endogeneity problem that arises in estimation, the authors propose a copula correction. To do so, they make use of an explicit model of the joint distribution of $(p_t, \xi_t)$. Using the model, the error term is decomposed into a part that is correlated with $p_t$ and a part that is not, and the structure of the model delivers a term—the copula correction term, a function of observable data—that controls for the part of the error term that is correlated with $p_t$.

Specifically, the authors assume that the latent pair $(p_t^*, \xi_t^*)$ follows a standard bivariate normal distribution with correlation coefficient $\rho$. The observed price and the latent variable are linked by

$$p_t^* = \Phi^{-1}(F_P(p_t)), \tag{2}$$

where $\Phi(\cdot)$ is the standard normal CDF and $F_P(\cdot)$ is the CDF of $p_t$. In principle, $p_t$ can be any monotonically increasing function of $p_t^*$: given $F_P(\cdot)$ and under the assumption that $p_t^*$ is standard normal, equation (2) holds as long as $p_t$ is a monotonically increasing function of $p_t^*$. Likewise, $\xi_t^* = \Phi^{-1}(F_\xi(\xi_t))$, where $F_\xi(\cdot)$ is the CDF of $\xi_t$; when $\xi_t$ is assumed normal, $\xi_t = \sigma_\xi \, \xi_t^*$.

Using a Cholesky decomposition, the distribution of $(p_t^*, \xi_t^*)$ can be rewritten as

$$\begin{pmatrix} p_t^* \\ \xi_t^* \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{pmatrix} \begin{pmatrix} \omega_{1t} \\ \omega_{2t} \end{pmatrix},$$

where $\omega_{1t}$ and $\omega_{2t}$ are orthogonal standard normal random variables. Using this representation, $\xi_t^* = \rho \, p_t^* + \sqrt{1-\rho^2}\, \omega_{2t}$, where we make use of $p_t^* = \omega_{1t}$. Replacing this expression in equation (1), and assuming $\xi_t$ is normally distributed (i.e., $\xi_t = \sigma_\xi \, \xi_t^*$), yields

$$q_t = \alpha_0 + \alpha_1 p_t + \sigma_\xi \cdot \rho \cdot p_t^* + \sigma_\xi \cdot \sqrt{1-\rho^2} \cdot \omega_{2t}. \tag{3}$$

In this expression, the part of $\xi_t$ that is correlated with $p_t$ is given by $\sigma_\xi \, \rho \, p_t^*$. Hence, including the copula correction term, $p_t^* = \Phi^{-1}(F_P(p_t))$, may alleviate the endogeneity problem arising from the correlation between $p_t$ and $\xi_t$: the remaining error $\omega_{2t}$ is independent of the regressors, and OLS applied to equation (3) identifies the parameters. Importantly, if $p_t$ is normally distributed, $p_t$ and $p_t^*$ are collinear—non-normality of $p_t$ is therefore required for the copula correction to help identify the coefficients of interest. Recent work has devoted considerable attention to this requirement and to the finite-sample consequences of weak non-normality (Eckert and Hohberger, 2023; Hu et al., 2025).

Our focus is different. In what follows, we take the population distribution of the data as known and ask under which data-generating processes for $p_t$ the augmented regression (3) identifies the structural parameters.

# 3 Identification in a single-equation data-generating process

We begin our analysis by characterizing a class of data-generating processes for which the copula correction point-identifies the structural parameters, formalizing the ideas in the previous section. To do so, we make explicit the conditions on which identification rests.

Consider the following data-generating process (DGP):

$$q_t = \alpha_0 + \alpha_1 p_t + \xi_t + \varepsilon_t \qquad \text{(structural equation)} \tag{4}$$

$$p_t = g(\xi_t, c_t) \qquad \text{(pricing equation)} \tag{5}$$

where $q_t$ and $p_t$ are observed, $\xi_t$ and $\varepsilon_t$ are unobserved, and $c_t$ is a degenerate constant, $\text{Var}(c_t) = 0$. The structural error has two components. The component $\xi_t$ enters the pricing equation: equation (5) nests, for example, the pricing rule of a monopolist facing a constant marginal cost, where the firm observes the demand shock $\xi_t$ and sets the price as a function of $\xi_t$ and the (constant) cost. The component $\varepsilon_t$ does not enter the pricing equation; two possible interpretations are a demand disturbance realized after prices are set and measurement error in the dependent variable. We maintain the following assumptions.

**Assumption 1** (Monotonicity in a single unobservable)**.** *$g(\xi_t, c_t)$ is strictly monotone in $\xi_t$.*

**Assumption 2** (Distributional knowledge)**.** *The marginal distribution of the error component that enters the pricing equation is known, continuous, and symmetric around zero: $\xi_t \sim F_\xi$.*

**Assumption 3** (Non-linearity)**.** *$g(\cdot, c_t)$ is strictly non-linear in $\xi_t$; that is, there exist no constants $(a, b)$ such that $g(\xi_t, c_t) = a + b\,\xi_t$ with probability one.*

**Assumption 4** (Exogenous noise)**.** *$\varepsilon_t$ has finite variance and $E[\varepsilon_t \mid \xi_t] = 0$.*

Importantly, Assumption 4 is the standard exogeneity requirement on an additive disturbance; it plays no role in the identification argument, and the degenerate case $\mathrm{Var}(\varepsilon_t) = 0$ is permitted.

The estimator is the population least-squares regression

$$q_t = \alpha_0 + \alpha_1 p_t + \gamma\, p_t^* + \nu_t, \qquad p_t^* \equiv F_\xi^{-1}(F_P(p_t))\,, \tag{6}$$

where $F_P$ and $F_\xi$ are the cumulative distribution functions of price and $\xi$, respectively, and $p_t^*$ corresponds to the copula correction under Assumption 2.

**Theorem 1.** *Under Assumptions 1–4, the parameters $(\alpha_0, \alpha_1, \gamma)$ in equation (6) are uniquely point-identified, with $\alpha_0$ and $\alpha_1$ equal to the structural parameters of equation (4) and with population residual $\nu_t = \varepsilon_t$.*

*Proof.* The proof proceeds in three steps.

*Step 1 (perfect recovery of the unobservable).* Because $c_t$ is degenerate at some value $\bar{c}$, write $p_t = h(\xi_t)$ with $h(\cdot) \equiv g(\cdot, \bar{c})$. By Assumption 1, $h$ is strictly monotone; suppose first that $h$ is strictly increasing. Then for any $t$ in the support of $p$,

$$F_P(t) = \Pr\left(h(\xi) \le t\right) = \Pr\left(\xi \le h^{-1}(t)\right) = F_\xi\left(h^{-1}(t)\right),$$

where the second equality uses strict monotonicity and the third uses Assumption 2. Evaluating at $t = p_t = h(\xi_t)$ gives $F_P(p_t) = F_\xi(\xi_t)$, and therefore

$$p_t^* = F_\xi^{-1}(F_P(p_t)) = F_\xi^{-1}(F_\xi(\xi_t)) = \xi_t.$$

If instead $h$ is strictly decreasing, the same argument gives $F_P(p_t) = 1 - F_\xi(\xi_t) = F_\xi(-\xi_t)$ (using that $F_\xi$ is symmetric around zero), so that $p_t^* = -\xi_t$. In either case, $p_t^*$ recovers $\xi_t$ exactly up to sign. In what follows we treat the increasing case—the decreasing case being identical with $\gamma = -1$ in place of $\gamma = 1$.

*Step 2 (the augmented regression holds with an exogenous residual).* Substituting $\xi_t = p_t^*$

into the structural equation (4) yields

$$q_t = \alpha_0 + \alpha_1 p_t + \gamma\, p_t^* + \nu_t, \qquad \text{with } \gamma = 1 \text{ and } \nu_t = \varepsilon_t.$$

By Assumption 4, $E[\varepsilon_t] = 0$; and because $p_t = h(\xi_t)$ and $p_t^* = \xi_t$ are functions of $\xi_t$ alone, the law of iterated expectations gives $E[\varepsilon_t\, p_t] = E[\, h(\xi_t)\, E(\varepsilon_t \mid \xi_t)\,] = 0$ and likewise $E[\varepsilon_t\, p_t^*] = 0$. The candidate coefficients $(\alpha_0, \alpha_1, 1)$, with the coefficients on the intercept and on $p_t$ equal to the structural parameters, therefore satisfy the orthogonality conditions of the population least-squares problem (6).

*Step 3 (rank condition).* It remains to show that $(\alpha_0, \alpha_1, \gamma)$ is the unique solution to the population least-squares problem, which requires that the regressors $(1, p_t, p_t^*)$ not be perfectly collinear. Suppose, toward a contradiction, that there exist constants $(a_0, a_1, a_2)$, not all zero, such that $a_0 + a_1 p_t + a_2 p_t^* = 0$ with probability one. Because $p_t^* = \xi_t$ has positive variance, it cannot be that $a_1 = 0$ (else $a_2 = 0$ and then $a_0 = 0$). Hence $p_t = -(a_0 + a_2 \xi_t)/a_1$, an affine function of $\xi_t$, contradicting Assumption 3. Therefore the second-moment matrix of $(1, p_t, p_t^*)$ is full rank, $|\mathrm{Corr}(p_t, p_t^*)| < 1$, and the population least-squares coefficients are unique. Combined with Step 2, where the candidate coefficients satisfy the orthogonality conditions with residual $\varepsilon_t$, the unique solution is $(\alpha_0, \alpha_1, 1)$, and the population residual is $\nu_t = \varepsilon_t$. $\square$

Theorem 1 establishes that the copula correction works in a specific class of data-generating processes: the correction term absorbs the endogenous component of the error exactly, and the population residual is the exogenous disturbance $\varepsilon_t$ alone.

The value of this result lies in what it makes explicit. Specifically, identification rests on three strict conditions. (Assumption 4 carries no identifying content.) First, a *single* unobservable drives the endogenous variable; the degeneracy of $c_t$ is what allows the marginal distribution of $p_t$ to be a one-to-one transformation of the marginal distribution of $\xi_t$. Second, the researcher knows the *correct marginal distribution* of $\xi_t$—the distribution of the error component that enters the pricing equation, not of the composite error $\xi_t + \varepsilon_t$; the equality $F_\xi^{-1}(F_P(p_t)) = \xi_t$ in Step 1 uses Assumption 2 directly. Third, the pricing equation is *strictly non-linear*, so that $p_t$ and $\xi_t$ are not identically distributed up to an affine transformation; non-linearity delivers the rank condition. Each of the three conditions is necessary for the argument, and Sections 4–6 examine what happens when the first condition fails.

The single-unobservable condition of Theorem 1 bears directly on the relationship between the copula correction and instrumental variables. The condition requires the price to be a strictly monotone function of the demand shock alone, $p_t = h(\xi_t)$. An excluded instrument for price is a variable that shifts the price while remaining independent of the structural error. No such variable can exist when $p_t = h(\xi_t)$: anything independent of $\xi_t$ leaves the price unchanged, and anything that moves the price moves $\xi_t$. The correction, therefore, identifies the structural parameter only when the price carries no exogenous variation—that is, only when no instrument is available.[1] The same observation explains the failures documented in Sections 5 and 6: when an excluded shifter does move the price, the price depends on a second unobservable, the conditions of Theorem 1 fail, and the copula method yields inconsistent estimates whether or not the shifter is observed. Exogenous price variation is what instrumental variables require and exactly what the copula's identifying assumption rules out.

A final remark concerns how our setup relates to that of Park and Gupta (2012). There, $p_t^*$ is a latent construct: the dependence between the endogenous regressor and the structural error is specified directly through the copula, rather than derived from a model of how the regressor is determined. A latent variable of this kind lacks a clear interpretation within an economic model. In an economic model, the endogenous variables are equilibrium functions of the exogenous variables and parameters. Because $\xi_t$ is a demand shifter, we instead specify the pricing equation (5), under which price is a function of $\xi_t$; the equality $p_t^* = \xi_t$ is then an implication of the model (Step 1 of the proof of Theorem 1) rather than an assumption. Exact recovery of $\xi_t$ is, moreover, stronger than what the method requires: Qian et al. (2026) argue that recovering $E[\xi_t \mid p_t]$ suffices for identification in their setting. The two coincide here ($E[\xi_t \mid p_t] = \xi_t$)—because $c_t$ is degenerate and $h$ is strictly monotone, $\xi_t$ is a deterministic function of $p_t$, so $E[\xi_t \mid p_t] = \xi_t$—but the two diverge once the price depends on more than one unobservable. In the remainder of the paper we therefore evaluate the correction term against the weaker criterion: the question is whether it recovers the conditional expectation of the structural error given price, up to scale, not whether it recovers the error itself.

## 3.1 Discussion: The Role of Parametric Assumptions

The copula method is parametric in two respects. First, the Gaussian copula assumes a specific dependence structure for $(p_t^*, \xi_t)$. Second, the construction of the correction term

[1] A finding that the copula method and an instrumental variable regression yield similar results would then imply invalidity of the instrument.

assumes a specific marginal distribution for $\xi_t$. Knowledge of the true marginal distribution of the shock is required for $F_\xi^{-1}(F_P(p_t)) = \xi_t$ to hold and, hence, for the correction term to solve the endogeneity problem exactly.

When $F_\xi^{-1}(F_P(p_t)) \neq \xi_t$, the augmented regression

$$q_t = \alpha_0 + \alpha_1 p_t + \gamma\, F_\xi^{-1}(F_P(p_t)) + \nu_t$$

has residual $\nu_t = \xi_t + \varepsilon_t - \gamma\, F_\xi^{-1}(F_P(p_t))$, which remains correlated with $p_t$ for any value of $\gamma$ unless $E[\xi_t|p_t]$ happens to be proportional to $F_\xi^{-1}(F_P(p_t))$. The discrepancy $F_\xi^{-1}(F_P(p_t)) \neq \xi_t$ can arise even when the copula itself is correctly specified—for example, when the researcher replaces $F_P$ with an estimate $\hat{F}_P$, as is done in every application. Park and Gupta (2012) argue on the basis of simulations that the normal distribution can approximate other marginal distributions of the shock, keeping the residual bias small. This is not true in general: the bias depends on the shape of $E[\xi_t \mid p_t]$ relative to the imposed correction term, which is governed by the data-generating process rather than by the quality of a density approximation. The simulations in Section 6 show that the bias can be of first-order magnitude in standard pricing models.

The more fundamental limitation, however, is not parametric. The next section shows that when the endogenous variable is determined in equilibrium by a system of equations with multiple unobservables, the structural parameters are not identified by *any* function of the endogenous variable—Gaussian or otherwise. The failure is therefore not one that flexible copulas, non-parametric estimates of $F_P$, or alternative marginal distributions can repair.

# 4 Simultaneous Equations

To examine the impact of simultaneity, consider two structural equations that jointly determine an equilibrium outcome:

$$q_t = f_D(p_t) + u_{Dt} \qquad \text{(demand)} \qquad (7)$$

$$q_t = f_S(p_t) + u_{St} \qquad \text{(supply)} \qquad (8)$$

where $f_D$ and $f_S$ are functions, $u_{Dt}$ and $u_{St}$ are mean-zero, non-degenerate structural errors, and $t$ indexes markets, as in the preceding sections. The pair $(p_t, q_t)$ is determined in equilibrium by the intersection of the two equations.

A key observation follows directly from the equilibrium condition that demand equals supply. Importantly, because $f_D(p_t) + u_{Dt} = f_S(p_t) + u_{St}$, we have that

$$u_{Dt} - u_{St} = f_S(p_t) - f_D(p_t). \tag{9}$$

The difference between the two structural errors is a deterministic function of the equilibrium price. Equation (9) encodes the simultaneity problem: the equilibrium price aggregates both unobservables, and the data on $(p_t, q_t)$ cannot disentangle them.

Before turning to identification formally, we describe how the copula correction operates in this environment. The augmented regression (6) regresses $q_t$ on $p_t$ and the single generated regressor $p_t^* = F_\xi^{-1}(F_P(p_t))$, a known function of $p_t$. Every regressor is therefore a function of $p_t$ alone, and the estimator seeks to recover the structural slope only through the conditional mean $E[q_t \mid p_t]$. Writing $m(p_t) \equiv E[u_{Dt} \mid p_t]$ for the conditional mean of the structural error—a feature of the data-generating process, in combination with the structure of the supply–demand model—the data deliver the single object

$$E[q_t \mid p_t] = f_D(p_t) + m(p_t).$$

A control-function estimator seeks to recover $f_D$ by supplying a regressor intended to stand in for $m(p_t)$—in the Gaussian copula correction, the term $\Phi^{-1}(F_P(p_t))$. The difficulty is that this decomposition is not unique: for any function $\delta(\cdot)$, the alternative pair $(f_D + \delta,\ m - \delta)$ reproduces $E[q_t \mid p_t]$ exactly; the conditional mean alone cannot identify which member is the truth. Even with the population distribution of $(p_t, q_t)$ in hand, the researcher cannot recover $m(p_t)$: it depends on the structure—the curves and the joint distribution of the unobservables—that the data leave undetermined, so the researcher must take a stance on $m(p_t)$. The copula correction is one such stance, and a generically misspecified one: nothing in the equilibrium structure makes $m(p_t)$ proportional to $\Phi^{-1}(F_P(p_t))$. Each particular stance on $m(p_t)$ selects one demand curve from the family $(f_D + \delta,\ m - \delta)$, all of which fit the data equally well, yet all of which (with at most one exception) depart from the truth. The propositions below establish that the true demand curve cannot be told apart from these alternatives using data on prices and quantities alone.

An alternative model counts against identification only if it both reproduces the joint distribution of the data and satisfies every restriction that the model class maintains, including market clearing and the slope restrictions on the two curves. We therefore make the class of structures and the identification concept explicit.

**Assumption 5** (Regular market equilibrium)**.** *$f_D$ is continuous and strictly decreasing, $f_S$ is continuous and strictly increasing, and the excess-supply function $f_S - f_D$ maps $\mathbb{R}$ onto $\mathbb{R}$.*

Under Assumption 5, for every $(u_D, u_S) \in \mathbb{R}^2$ the market-clearing condition $f_D(p) + u_D = f_S(p) + u_S$ has a unique solution, $p = (f_S - f_D)^{-1}(u_D - u_S)$; write $\pi_{(f_D,f_S)}(u_D, u_S)$ for this equilibrium map. A *structure* is a triple $S = (f_D, f_S, F)$ for the system (7)–(8), where the pair $(f_D, f_S)$ satisfies Assumption 5 and $F$ is a joint distribution for $(u_{Dt}, u_{St})$ with mean-zero, non-degenerate marginals; beyond these conditions, $F$ is unrestricted—in particular, nothing ties the demand shock to the supply shock.[2] A structure generates the data by drawing, in each market $t$, $(u_{Dt}, u_{St}) \sim F$, setting $p_t = \pi_{(f_D,f_S)}(u_{Dt}, u_{St})$, and setting $q_t = f_D(p_t) + u_{Dt}$.

Let $\mathcal{S}_{\text{SD}}$ denote the class of all structures. Two structures are *observationally equivalent* if they induce the same joint distribution of $(p_t, q_t)$, which is the researcher's entire information set in the absence of instruments, and $f_D$ is *point-identified* at $S_0 \in \mathcal{S}_{\text{SD}}$ if every structure in $\mathcal{S}_{\text{SD}}$ that is observationally equivalent to $S_0$ shares its demand function.

**Proposition 1** (Non-identification I)**.** *Fix a structure $S_0 = (f_D, f_S, F) \in \mathcal{S}_{\text{SD}}$. Then $f_D$ is not point-identified from the joint distribution of $(p_t, q_t)$: there is a continuum of structures in $\mathcal{S}_{\text{SD}}$, with demand functions distinct from $f_D$, that are observationally equivalent to $S_0$.*

*Proof.* The proof proceeds in four steps. Throughout, $(u_{Dt}, u_{St}) \sim F$ are the primitives of $S_0$, and $p_t = \pi_{(f_D,f_S)}(u_{Dt}, u_{St})$ and $q_t = f_D(p_t) + u_{Dt}$ are the realized equilibrium price and quantity. Fix $\lambda < 0$.

*Step 1 (the perturbed structure).* Define the perturbed demand function and, on the same probability space, the perturbed shocks

$$\tilde{f}_D \equiv f_D + \lambda\,(f_S - f_D), \qquad \tilde{u}_{Dt} \equiv (1-\lambda)\,u_{Dt} + \lambda\,u_{St}, \qquad \tilde{u}_{St} \equiv u_{St},$$

which leave the supply function unchanged. Let $\tilde{F}$ be the joint distribution of $(\tilde{u}_{Dt}, \tilde{u}_{St})$, and set $\tilde{S} \equiv (\tilde{f}_D, f_S, \tilde{F})$.

[2] The mean-zero marginals are a location normalization. Leaving the dependence between $u_{Dt}$ and $u_{St}$ unrestricted matches the information set of instrument-free applications, which bring no knowledge of how demand and supply shocks co-move.

*Step 2 (the realized data satisfy both perturbed equations).* At the realized $(p_t, q_t)$,

$$\tilde{f}_D(p_t) + \tilde{u}_{Dt} = (1-\lambda)\Big(f_D(p_t) + u_{Dt}\Big) + \lambda\Big(f_S(p_t) + u_{St}\Big) = (1-\lambda)\, q_t + \lambda\, q_t = q_t,$$

and the supply side is untouched: $q_t = f_S(p_t) + \tilde{u}_{St}$.

*Step 3 (the realized price is the perturbed equilibrium).* The perturbed excess-supply function is $f_S - \tilde{f}_D = (1-\lambda)(f_S - f_D)$. Because $\lambda < 0$, the function $\lambda\,(f_S - f_D)$ is continuous and strictly decreasing, so $\tilde{f}_D$ is continuous and strictly decreasing, and $f_S - \tilde{f}_D$ is continuous, strictly increasing, and onto $\mathbb{R}$: the pair $(\tilde{f}_D, f_S)$ satisfies Assumption 5, and the perturbed market-clearing condition has a unique solution for every realization of the shocks. By Step 2, the realized price solves it, so $\pi_{(\tilde{f}_D, f_S)}(\tilde{u}_{Dt}, \tilde{u}_{St}) = p_t$ almost surely.

*Step 4 (admissibility and equivalence).* First, $E[\tilde{u}_{Dt}] = (1-\lambda)\,E[u_{Dt}] + \lambda\,E[u_{St}] = 0$ and $E[\tilde{u}_{St}] = 0$: the location normalization is preserved exactly. Second, $\mathrm{Var}(\tilde{u}_{St}) = \mathrm{Var}(u_{St}) > 0$, and $\mathrm{Var}(\tilde{u}_{Dt}) > 0$ for all but at most one value of $\lambda$,[3] so a continuum of values of $\lambda < 0$ delivers $\tilde{S} \in \mathcal{S}_{\mathrm{SD}}$. Third, $\tilde{f}_D - f_D = \lambda\,(f_S - f_D)$ is not identically zero, because $\lambda \neq 0$ and $f_S - f_D$ is onto $\mathbb{R}$. Fourth, by Steps 2 and 3, the observables generated by $\tilde{S}$ from the primitives $(\tilde{u}_{Dt}, \tilde{u}_{St})$ coincide with the observables generated by $S_0$ from $(u_{Dt}, u_{St})$ realization by realization; because the observables of a structure are a deterministic function of its primitives—the equilibrium map followed by the demand equation—the two structures induce the same joint distribution of $(p_t, q_t)$. Every admissible $\lambda$ thus yields a structure in $\mathcal{S}_{\mathrm{SD}}$ with a demand function distinct from $f_D$ that is observationally equivalent to $S_0$, and $f_D$ is not point-identified. $\square$

At least two features of the construction are worth emphasizing. First, the alternative is itself a well-specified supply–demand model: the perturbed demand curve slopes down, the supply side—both the curve and the shock—is untouched, and the perturbed shocks satisfy the location normalization exactly. Non-identification is therefore not an artifact of admitting ill-behaved alternatives. Second, the equivalence holds at the level of the full joint distribution of $(p_t, q_t)$, realization by realization, not merely the conditional mean: no statistic computed from the data can distinguish $\tilde{S}$ from $S_0$. At the level of conditional means, every pair $(\tilde{f}_D, \tilde{m}) = (f_D + \delta,\ m - \delta)$, with $m(p_t) \equiv E[u_{Dt} \mid p_t]$ and $\delta(\cdot)$ an arbitrary function, reproduces $E[q_t \mid p_t]$. The copula correction selects one member of this family. Imposing the Gaussian-copula structure amounts to asserting that $E[u_{Dt} \mid p_t]$

[3] $\tilde{u}_{Dt} = u_{Dt} + \lambda\,(u_{St} - u_{Dt})$ is affine in $\lambda$, so $\mathrm{Var}(\tilde{u}_{Dt})$ is a quadratic in $\lambda$. Being a variance it is non-negative, and it equals $\mathrm{Var}(u_{Dt}) > 0$ at $\lambda = 0$; a non-negative quadratic that is positive at a point vanishes at most once, so $\mathrm{Var}(\tilde{u}_{Dt}) > 0$ for all but at most one $\lambda$.

is proportional to $\Phi^{-1}(F_P(p_t))$—an assertion that the equilibrium structure of the model does not deliver and that the data cannot verify.

Proposition 1 establishes non-identification at the level of structures. We next work at the level of the object the copula correction estimates—a regression of $q_t$ on functions of $p_t$—and make the consequence of equation (9) concrete. We show that not only is the demand function not point-identified, but we find that the conditional expectation of quantity given price is *identical* whether one writes it in terms of the demand equation or the supply equation.

**Proposition 2** (Non-identification II)**.** *Fix a structure $S_0 = (f_D, f_S, F) \in \mathcal{S}_{\mathrm{SD}}$. Demand and supply cannot be separately identified by a regression of $q_t$ on functions of $p_t$ alone. In particular, the copula correction does not identify $f_D$.*

*Proof.* The proof proceeds in four steps.

*Step 1.* From the equilibrium condition, $u_{Dt} - u_{St} = f_S(p_t) - f_D(p_t)$.

*Step 2.* Rearranging and taking expectations conditional on $p_t$ (the right-hand side of Step 1 is measurable with respect to $p_t$),

$$E[u_{Dt} \mid p_t] = f_S(p_t) - f_D(p_t) + E[u_{St} \mid p_t].$$

*Step 3.* The conditional expectation of quantity given price, written from the demand side, is

$$\begin{aligned} E[q_t \mid p_t] &= f_D(p_t) + E[u_{Dt} \mid p_t] \\ &= f_D(p_t) + f_S(p_t) - f_D(p_t) + E[u_{St} \mid p_t] \\ &= f_S(p_t) + E[u_{St} \mid p_t], \end{aligned}$$

where the second line substitutes the expression from Step 2.

*Step 4.* The conditional expectation of quantity given price, written from the supply side, is $E[q_t \mid p_t] = f_S(p_t) + E[u_{St} \mid p_t]$, which coincides with the expression in Step 3. The two structural equations therefore generate one and the same conditional expectation function, and no transformation of $p_t$ included as a regressor can attribute $E[q_t \mid p_t]$ to one equation rather than the other. □

The most the researcher can learn from the data is $E[q_t \mid p_t]$, but

$$E[q_t \mid p_t] = f_D(p_t) + E[u_{Dt} \mid p_t] = f_S(p_t) + E[u_{St} \mid p_t],$$

and the researcher knows neither $E[u_{Dt} \mid p_t]$ nor $E[u_{St} \mid p_t]$. It follows that no regression of $q_t$ on functions of $p_t$ alone—the copula term $\Phi^{-1}(F_P(p_t))$, the raw CDF $F_P(p_t)$, a polynomial in $p_t$, or any other transformation—can distinguish demand from supply. Section 6.1 illustrates Propositions 1 and 2 with simulations.

## 4.1 How an Instrument Breaks Observational Equivalence

To clarify what is missing in the instrument-free approach, we characterize how an excluded instrument resolves the observational-equivalence problem; the characterization makes precise the source of identifying power that the copula correction lacks. We focus here on the case in which $f_D$ is linear to match the canonical specification of equation (1) and for expositional purposes.[4]

Recall the observationally equivalent family. Within the linear class, an alternative demand function differs from $f_D$ by an affine perturbation $\delta(p_t) = \delta_0 + \delta_1 \, p_t$:

$$\tilde{f}_D(p_t) = f_D(p_t) + \delta_0 + \delta_1 \, p_t, \qquad \tilde{u}_{Dt} = u_{Dt} - \delta_0 - \delta_1 \, p_t,$$

and the location normalization $E[\tilde{u}_{Dt}] = 0$ pins down the constant component at $\delta_0 = -\delta_1 \, E[p_t]$. The family is therefore one-dimensional, indexed by the slope perturbation $\delta_1$, and identifying $f_D$ requires ruling out every $\delta_1 \neq 0$. The structures constructed in the proof of Proposition 1 fall inside this family when the supply curve is linear as well: $\delta(p_t) = \lambda \left( f_S(p_t) - f_D(p_t) \right)$ is then affine in $p_t$, with slope perturbation equal to $\lambda$ times the difference between the supply and demand slopes.

Introduce an excluded supply shifter $Z_t$ satisfying exogeneity, $E[u_{Dt} \mid Z_t] = 0$, and relevance in the conditional-mean sense: $E[p_t \mid Z_t]$ is non-constant in $Z_t$. For an alternative in the family to be valid, its implied structural error must also satisfy exogeneity with

[4] The characterization extends beyond the linear class at the cost of a stronger condition on the instrument. With $f_D$ treated non-parametrically, the observationally equivalent family of Proposition 2 is indexed by an *arbitrary* perturbation $\delta(\cdot)$, with $\tilde{f}_D = f_D + \delta$ and $\tilde{u}_{Dt} = u_{Dt} - \delta(p_t)$, and the argument below shows that validity of an alternative requires $E[\delta(p_t) \mid Z_t = z] = 0$ for almost every $z$. Ruling out every non-zero perturbation is then precisely *completeness* of the conditional distribution of $p_t$ given $Z_t$—the identification condition of non-parametric instrumental-variables models (Newey and Powell, 2003)—which is strictly stronger than relevance.

respect to $Z_t$. Substituting $\delta_0 = -\delta_1\, E[p_t]$,

$$E[\tilde{u}_{Dt} \mid Z_t] = E\Big[\, u_{Dt} - \delta_1 \left(p_t - E[p_t]\right) \mid Z_t \,\Big] = -\delta_1 \left(E[p_t \mid Z_t] - E[p_t]\right),$$

where the second equality uses $E[u_{Dt} \mid Z_t] = 0$. Validity of the alternative model therefore requires

$$\delta_1 \left(E[p_t \mid Z_t = z] - E[p_t]\right) = 0 \quad \text{for almost every } z. \tag{10}$$

Because $E[p_t \mid Z_t]$ is non-constant, condition (10) forces $\delta_1 = 0$: a single excluded shifter eliminates the entire observationally equivalent family, and $f_D$ is point-identified within the linear class. Exogeneity and conditional-mean relevance are, in the linear model, exactly what identification requires.

This characterization clarifies the contrast with the copula correction. An instrument supplies moment restrictions indexed by the values of an *excluded* variable—one restriction per value of $z$—and in the linear case these restrictions suffice to eliminate the one-dimensional family of perturbations. The copula correction supplies no such restrictions: the correction term is a fixed function of $p_t$ itself, so it cannot rule out any member of the family of Propositions 1 and 2.

# 5 Oligopoly Games

In this section, we show that the non-identification result extends to oligopoly games. We illustrate this in the context of an oligopoly pricing game where a demand shock enters the structural demand equation, and prices are determined by a pricing equation derived from firm optimization. As we develop in detail in Section 6, this is a common feature in standard models of monopoly and oligopoly pricing.

The pricing equations of those models violate Assumption 1 of Theorem 1: the endogenous variable is driven by more than one unobservable—marginal cost and the demand shock in the monopoly case, and the full vector of demand shocks (and costs) in the oligopoly case. The following proposition establishes that this violation precludes point identification. As in Section 4, an alternative structure counts against identification only if it reproduces the joint distribution of the data while satisfying every restriction the model class maintains—here including equilibrium behavior on the supply side—and we make the class and the identification concept explicit for this environment before stating the result.

Consider the following data-generating process, the equilibrium counterpart of the single-equation process (4)–(5). For $j = 1, \dots, J$,

$$f_j(q_j, q_0) = \alpha + \beta P_j + \xi_j \qquad \text{(structural equation)} \qquad (11)$$

$$P_j = C_j - \frac{1}{\beta\,(1 - s_j)} \qquad \text{(pricing equation)} \qquad (12)$$

where demand is multinomial logit with a linear price index: mean utilities are $v_j \equiv \alpha + \beta P_j + \xi_j$ with $\beta < 0$, shares are $s_j(\mathbf{P}, \boldsymbol{\xi}) = \exp\{v_j\}/\left(1 + \sum_{k=1}^{J} \exp\{v_k\}\right)$, and $f_j(q_j, q_0) \equiv \ln s_j - \ln s_0$ is the inverse of the share system (Berry, 1994). The pricing equation (12) is the first-order condition of a single-product firm that observes $(\boldsymbol{\xi}, \mathbf{C})$ and sets $P_j$ to maximize $(P_j - C_j)\, s_j$ given rivals' prices; we suppress the market index. We maintain the following assumption.

**Assumption 6** (Regular pricing game)**.** *For every $\beta < 0$ and every $(\boldsymbol{\xi}, \mathbf{C}) \in \mathbb{R}^J \times \mathbb{R}^J$, the Bertrand–Nash pricing game has a unique equilibrium, and the system of first-order conditions* (12) *characterizes it.*

Assumption 6 holds in the logit environment: each firm's profit is strictly pseudo-concave in its own price—the derivative of log profit, $1/(P_j - C_j) + \beta\,(1 - s_j)$, is strictly decreasing on $(C_j, \infty)$—so the first-order conditions characterize best responses, and the pricing game has a unique equilibrium for any $\beta < 0$ and any realization of shocks and costs (see, e.g., Nocke and Schutz (2018)). Write $\mathbf{g}_\beta(\boldsymbol{\xi}, \mathbf{C})$ for the resulting equilibrium price vector.

As in Section 4, it is useful to locate the copula correction within this environment before turning to identification formally. Applied to the logit model, the correction regresses the inverted share $f_j(q_j, q_0) = \ln s_j - \ln s_0$ on the price $P_j$ and the generated regressor $\Phi^{-1}(F_{P_j}(P_j))$, a known function of the firm's own price. Every regressor is therefore a function of prices, and the estimator seeks to recover the price coefficient only through the conditional mean $E[f_j(q_j, q_0) \mid \mathbf{P}]$. Writing $m_j(\mathbf{P}) \equiv E[\xi_j \mid \mathbf{P}]$ for the conditional mean of the demand shock—a feature of the data-generating process, in combination with the structure of the pricing game—the data deliver the single object

$$E[f_j(q_j, q_0) \mid \mathbf{P}] = \alpha + \beta P_j + m_j(\mathbf{P}).$$

A control-function estimator seeks to recover $\beta$ by supplying a regressor intended to stand in for $m_j(\mathbf{P})$—in the Gaussian copula correction, the term $\Phi^{-1}(F_{P_j}(P_j))$. The difficulty is again that this decomposition is not unique: for any $\delta$, the alternative pair $(\beta + \delta,\ m_j - \delta P_j)$—a price coefficient $\beta + \delta$ together with the conditional error mean

$m_j(\mathbf{P}) - \delta P_j$—reproduces $E[f_j(q_j, q_0) \mid \mathbf{P}]$ exactly, so the conditional mean alone cannot say which member is the truth. Even with the population distribution of $(\mathbf{P}, \mathbf{q})$ in hand, the researcher cannot recover $m_j(\mathbf{P})$: it depends on the structure—the price coefficient and the joint distribution of demand shocks and costs $(\boldsymbol{\xi}, \mathbf{C})$—that the data leave undetermined, so a stance on $m_j(\mathbf{P})$ is unavoidable. The copula correction is one such stance, and a generically misspecified one: nothing in the Bertrand–Nash structure makes $m_j(\mathbf{P})$ proportional to $\Phi^{-1}(F_{P_j}(P_j))$. Each choice selects one value of the price coefficient from this family, all of which fit the data equally well yet depart from the truth—and, as the proof below makes precise, each corresponds to a different division of observed prices into markup and marginal cost. The proposition that follows establishes that the true price coefficient cannot be told apart from these alternatives using data on prices and quantities alone.

In this environment, a *structure* is a triple $S = (\alpha, \beta, F)$, where $\beta < 0$ and $F$ is a joint distribution for $(\boldsymbol{\xi}, \mathbf{C})$ with $\mathrm{Var}(\xi_j) > 0$ and $\mathrm{Var}(C_j) > 0$ for all $j$; beyond these variance conditions, $F$ is unrestricted—in particular, the dependence between demand shocks and costs is unrestricted.[5] A structure generates the data by drawing $(\boldsymbol{\xi}, \mathbf{C}) \sim F$, setting $\mathbf{P} = \mathbf{g}_\beta(\boldsymbol{\xi}, \mathbf{C})$, and setting shares $s_j = s_j(\mathbf{P}, \boldsymbol{\xi})$—equivalently, quantities, for a known market size. Let $\mathcal{S}_{\mathrm{BN}}$ denote the class of all structures (the Bertrand–Nash counterpart of the supply–demand class $\mathcal{S}_{\mathrm{SD}}$ of Section 4). Two structures are *observationally equivalent* if they induce the same joint distribution of prices and quantities, which is the researcher's entire information set in the absence of instruments, and $\beta$ is *point-identified* at $S_0 \in \mathcal{S}_{\mathrm{BN}}$ if every structure in $\mathcal{S}_{\mathrm{BN}}$ that is observationally equivalent to $S_0$ shares its value of $\beta$.

**Proposition 3** (Non-identification III). *Let Assumption 6 hold, and fix a structure $S_0 = (\alpha, \beta, F) \in \mathcal{S}_{\mathrm{BN}}$. Then $\beta$ is not point-identified from the joint distribution of quantities and prices: there is a continuum of structures in $\mathcal{S}_{\mathrm{BN}}$, with price coefficients distinct from $\beta$, that are observationally equivalent to $S_0$.*

*Proof.* The proof proceeds in four steps. Throughout, $(\boldsymbol{\xi}, \mathbf{C}) \sim F$ are the primitives of $S_0$, and $\mathbf{P} = \mathbf{g}_\beta(\boldsymbol{\xi}, \mathbf{C})$ and $\mathbf{s}$ are the realized equilibrium prices and shares. Fix $\delta \in (-\infty, -\beta) \setminus \{0\}$ and set $\tilde{\beta} \equiv \beta + \delta < 0$.

*Step 1 (the perturbed structure).* On the same probability space, define

$$\tilde{\xi}_j \equiv \xi_j - \delta P_j, \qquad \tilde{C}_j \equiv P_j + \frac{1}{\tilde{\beta}\,(1 - s_j)}, \qquad j = 1, \ldots, J,$$

[5] Leaving the joint distribution of $(\boldsymbol{\xi}, \mathbf{C})$ unrestricted is deliberate: it is the class that applications without cost data maintain.

let $\tilde{F}$ be the joint distribution of $(\tilde{\boldsymbol{\xi}}, \tilde{\mathbf{C}})$, and set $\tilde{S} \equiv (\alpha, \tilde{\beta}, \tilde{F})$. Using the pricing equation (12) of $S_0$, the perturbed costs satisfy

$$\tilde{C}_j = C_j + \frac{\delta}{\tilde{\beta}} \Big( P_j - C_j \Big); \tag{13}$$

that is, the perturbation reclassifies the fraction $\delta/\tilde{\beta}$ of each firm's realized markup as marginal cost.

*Step 2 (the demand side is unchanged at the realized prices).* By construction, $\alpha + \tilde{\beta} P_j + \tilde{\xi}_j = \alpha + \beta P_j + \xi_j = v_j$ for every $j$: at the realized prices, mean utilities—and therefore shares—are identical under the two structures, and the structural equation (11) holds identically at the realized data with $(\tilde{\beta}, \tilde{\xi}_j)$ in place of $(\beta, \xi_j)$.

*Step 3 (the realized prices are the perturbed equilibrium).* Logit price derivatives depend on the price coefficient and the share levels only; because the share levels at $\mathbf{P}$ coincide across the two structures by Step 2, the own-price derivative of firm $j$'s share under $\tilde{S}$, evaluated at the realized prices, is $\tilde{\beta}\, s_j(1 - s_j)$. Firm $j$'s first-order condition under $(\tilde{\beta}, \tilde{\boldsymbol{\xi}}, \tilde{\mathbf{C}})$, evaluated at the realized prices, is therefore

$$s_j + \Big( P_j - \tilde{C}_j \Big)\, \tilde{\beta}\, s_j(1 - s_j) = 0,$$

which holds by the construction of $\tilde{C}_j$. By Assumption 6, the first-order conditions characterize the unique equilibrium of the pricing game, so $\mathbf{g}_{\tilde{\beta}}(\tilde{\boldsymbol{\xi}}, \tilde{\mathbf{C}}) = \mathbf{P}$ almost surely: the observed prices are exactly the Bertrand–Nash equilibrium prices of the perturbed structure.

*Step 4 (admissibility and equivalence).* First, $\tilde{\beta} < 0$ by the choice of $\delta$. Second, $\mathrm{Var}(\tilde{\xi}_j) > 0$ and $\mathrm{Var}(\tilde{C}_j) > 0$ for all $j$ for all but finitely many values of $\delta$, so that a continuum of values of $\delta$ delivers $\tilde{S} \in \mathcal{S}_{\mathrm{BN}}$.[6] Third, by Steps 2 and 3, the observables generated by $\tilde{S}$ from the primitives $(\tilde{\boldsymbol{\xi}}, \tilde{\mathbf{C}})$ coincide with the observables generated by $S_0$ from $(\boldsymbol{\xi}, \mathbf{C})$ realization by realization; because the observables of a structure are a deterministic function of its primitives—the equilibrium map of Assumption 6 followed by the demand system—the two structures induce the same joint distribution of $(\mathbf{P}, \mathbf{q})$. Every admissible $\delta$ thus yields a structure in $\mathcal{S}_{\mathrm{BN}}$ with price coefficient $\tilde{\beta} \neq \beta$ that is observationally equivalent to $S_0$, and $\beta$ is not point-identified. □

[6] A similar argument as the one in footnote 3 applies. Each variance condition is a non-negative quadratic that is positive at a point: the variance condition can equal zero at most for one value of $\delta$ per product. The exceptional set thus contains at most $2J$ points.

At least three features of Proposition 3 are worth emphasizing. First, the alternative structure is another linear logit demand model with a different price coefficient, supported by Bertrand–Nash pricing behavior, and the non-identification concerns the parameter that applied researchers report. Second, the observational equivalence holds at the level of the full joint distribution of prices and quantities, not merely the conditional mean: no statistic computed from $(\mathbf{P}, \mathbf{q})$ can distinguish the alternative from the truth. The result therefore holds for any choice of correction term that is a function of prices alone—the copula term $\Phi^{-1}(F_{P_j}(P_j))$, correction terms in all $J$ prices, a polynomial in prices, or any other transformation. The copula correction constructs one function of prices per endogenous regressor; the equilibrium pricing system embeds at least two unobservables per product.[7] Third, the equivalence has a direct economic interpretation. Equation (13) shows that the data on prices and quantities cannot separate price sensitivity, markups, and costs: a less elastic demand curve, larger markups, and lower costs fit the data exactly as well as the truth. This attribution problem is familiar from the literatures on the identification of conduct (Bresnahan, 1982) and of demand from market-level data (Berry and Haile, 2014); the proposition shows that the copula correction does not escape it.

The proof also makes precise where identifying power must come from. Nothing in $\mathcal{S}_{\text{BN}}$ ties the cost shocks to the demand shocks. Restrictions that link the two shrink the observationally equivalent family. Observed cost data pin down $\mathbf{C}$ directly; an excluded cost shifter supplies moment restrictions indexed by an exogenous variable (Section 5.1); and a covariance restriction such as $\text{Cov}(\xi_j, C_j) = 0$ (MacKay and Miller, 2025) is generically violated by the constructed alternative, because the cost adjustment in equation (13) is an equilibrium object and hence correlated with $\tilde{\xi}_j$. Proposition 3 therefore delineates the class that applications of the copula correction to pricing maintain, implicitly: no cost data, no exclusion restrictions, and no restrictions on the dependence between demand and cost shocks. Within that class, the division of prices between markup and cost is unrestricted, and so is the price coefficient.

The construction is not specific to the environment above. With multiproduct ownership and demand Jacobian $\Delta(\mathbf{P})$, the first-order conditions give $\mathbf{C} = \mathbf{P} + \Delta(\mathbf{P})^{-1}\mathbf{s}$; for any demand system with a linear price index, the Jacobian evaluated at the realized prices scales under the perturbation as $\tilde{\Delta} = (\tilde{\beta}/\beta)\,\Delta$, because mean utilities are unchanged, and equation (13) carries over without modification. More generally, the argument uses the

[7] At the level of conditional means, the equivalence implies $E[f_j(q_j, q_0) \mid \mathbf{P}] = \alpha + \tilde{\beta} P_j + \tilde{m}_j(\mathbf{P})$ with $\tilde{m}_j(\mathbf{P}) = m_j(\mathbf{P}) - \delta P_j$, where $m_j(\mathbf{P}) \equiv E[\xi_j \mid \mathbf{P}]$; this is the expression used in Section 5.1.

logit form only through Assumption 6 and the linearity of the price index, and it applies to any invertible demand system with these properties.[8] Section 6.2 illustrates Proposition 3 with simulations.

## 5.1 How an Instrument Breaks Equivalence

As in Section 4.1, the role of an instrument can be characterized exactly. The argument parallels the linear case there: every alternative structure constructed in the proof of Proposition 3 implies the structural error $\tilde{\xi}_j = \xi_j - \delta P_j$, so the observationally equivalent family is one-dimensional, indexed by $\delta$.

The alternative model shifts the parameter ($\tilde{\beta} = \beta + \delta$) and forces an implied structural error to match the data:

$$\tilde{\xi}_j = \xi_j - \delta P_j.$$

Introduce an excluded cost shifter $Z$ (e.g., a vector of raw-material prices) satisfying exogeneity, $E[\xi_j \mid Z] = 0$, and relevance, in the sense that $E[P_j \mid Z]$ is non-constant in $Z$. For the alternative parameter $\tilde{\beta}$ to be valid, its implied error must also be exogenous with respect to $Z$:

$$E[\tilde{\xi}_j \mid Z] = E[\, \xi_j - \delta P_j \mid Z\,] = -\delta\, E[\, P_j \mid Z\,],$$

where the second equality uses $E[\xi_j \mid Z] = 0$. Because $E[P_j \mid Z]$ is non-constant in $Z$, the right-hand side is zero for almost every value of $Z$ only if $\delta = 0$.[9] The instrument thus identifies $\beta$. The contrast with the copula correction is the same as before: the instrument supplies moment restrictions indexed by an excluded variable, and these restrictions eliminate every alternative model in the observationally equivalent family; a correction term built from prices alone supplies no such restrictions.

[8] The class $\mathcal{S}_{\mathrm{BN}}$ imposes no location normalization on $\boldsymbol{\xi}$. Under the normalization $E[\xi_j] = 0$, replace $\tilde{\xi}_j$ with $\xi_j - \delta\,(P_j - E[P_j])$ and absorb $\delta\, E[P_j]$ into product-specific intercepts; with the single common intercept of equation (11), exact re-centering requires $E[P_j]$ to be common across products, as in the symmetric duopoly of Section 6.2.

[9] If $E[P_j \mid Z]$ were constant—instruments that are exogenous but irrelevant—the displayed condition would hold for a continuum of values of $\delta$ after re-centering, and identification would fail. Note also that, in contrast to the non-parametric case discussed in footnote 4 of Section 4.1, non-constancy of the conditional mean suffices here: the perturbation is one-dimensional, so a completeness condition on the full conditional distribution is not required.

# 6 Examples and Simulation Evidence

In economic models, endogenous variables are typically determined by equilibrium systems with multiple unobservables. These variables rarely follow an exogenous joint distribution with a single error term. To evaluate whether the copula correction delivers unbiased estimates in these settings, we present three examples covering four classic economic environments.

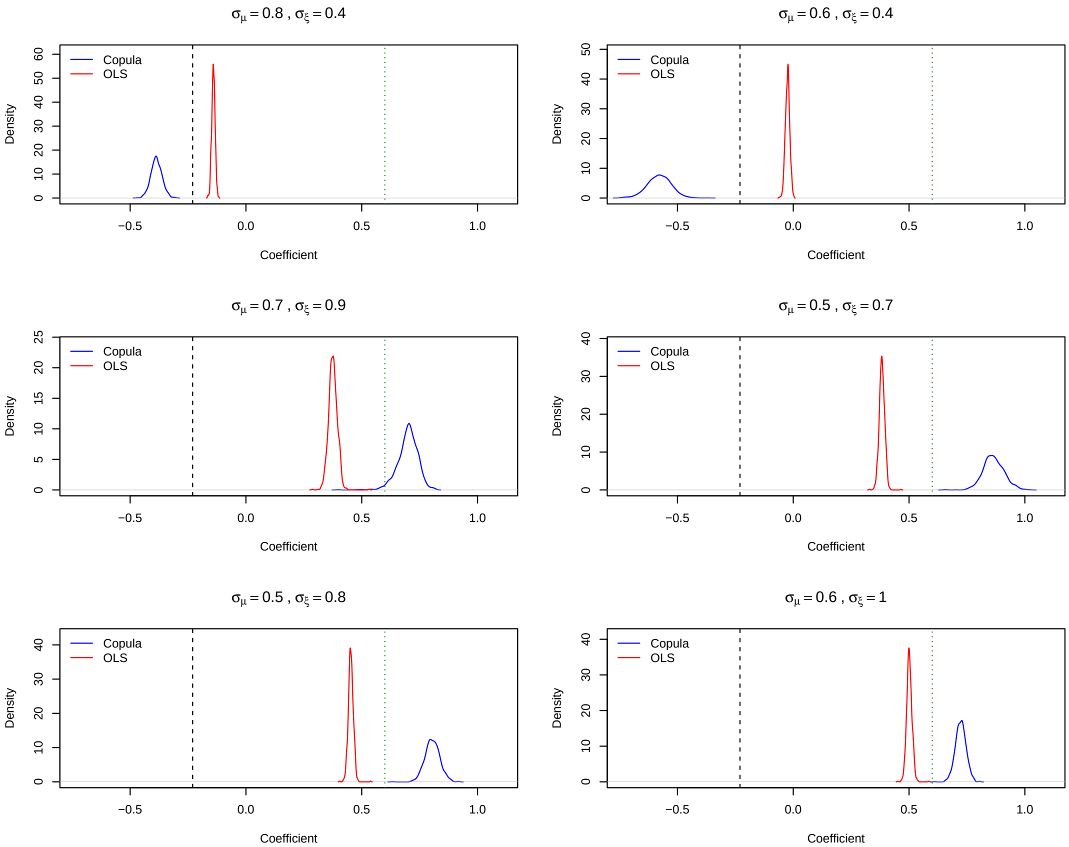
In what follows, we estimate the equation of interest across all examples using OLS with the Gaussian copula correction term $\Phi^{-1}(\hat{F}_P(\cdot))$, computing coefficients across 1,000 samples of 5,000 observations each. Computing the Gaussian copula correction requires an estimate of $\hat{F}_P(p) \in (0,1)$ for every value of $p$ in the sample. To ensure that this is the case, we estimate $\hat{F}_P(\cdot)$ using the recommendation in Qian et al. (2026). Given a sample of $n$ prices sorted from smallest to largest, $p_1 \leq p_2 \leq \cdots \leq p_n$, let $\hat{F}(p_i) = i/n$ for $i < n$, and $\hat{F}(p_n) = n/(n+1)$.

Throughout these examples, we report the index of copula-model non-identification (ICON) proposed by Qian et al. (2026)—defined as the ratio of the parameter's copula-corrected standard error to its uncorrected OLS standard error. This index serves as a diagnostic tool to identify scenarios where copula correction is inappropriate or requires refinement, such as when the copula term is nearly collinear with the endogenous variable. Specifically, the authors suggest that an ICON value exceeding six flags the model as potentially weakly identified or misspecified, indicating a need for adjustment. The free parameters in the data generating processes in our Monte Carlo exercises are selected to keep the ICON scores below six. We find that scores below six are not sufficient to give good properties to the copula method.

## 6.1 Example 1: Supply and Demand

We first consider a supply and demand setting. Estimating supply and demand curves is a classic simultaneity problem (Roberts and Schlenker, 2013; Asker et al., 2019). We evaluate whether the copula correction can substitute for an excluded instrument in this setting.

**Figure 1:** Estimated coefficients for Example 1 using the copula method and OLS for different values of $\sigma_\xi$ and $\sigma_\mu$




Notes: The left-most vertical line is positioned at $x = -0.23$ (demand slope), whereas the right-most vertical line is positioned at $x = 0.6$ (supply slope). All six parameter configurations have an ICON score of less than 6 (Qian et al., 2026).

**Model and Econometrics.** We simulate a standard linear system:

$$
\begin{aligned}
q_t^d &= 5 - 0.23p_t + \xi_t, \\
q_t^s &= 2 + 0.60p_t + \mu_t, \qquad (14)
\end{aligned}
$$

where the demand and supply shocks, $\xi_t$ and $\mu_t$, are uniformly distributed, $\xi_t \sim \log N(-1, \sigma_\xi^2)$ and $\mu_t \sim \log N(-1, \sigma_\mu^2)$, and the parameters on both curves are arbitrary. The equilibrium price $p_t$ is determined by both primitive variables ($\xi_t$ and $\mu_t$). We assume the econometrician estimates one curve or the other.

**Results.** We find that the copula method fails to recover the slope of either structural curve, as does OLS, as shown in Figure 1. On the one hand, the OLS estimates yield a weighted average of the demand and supply curves, with weights that depend on the relative magnitudes of the variances of supply and demand shocks: when most variation is on the supply side, the estimate is closer to the slope of the demand curve. On the other hand, the copula method may yield a slope that is more negative than the demand curve slope or more positive than the supply curve slope. This implies that the copula method may yield estimates that are farther from the truth than OLS for both curves. We also note that all parameter configurations in Figure 1 have an ICON value of less than six, showing that the poor performance of the copula method holds even when the ICON diagnostic does not flag the application as problematic Qian et al. (2026).

Consistent with Propositions 1 and 2, without an external instrument, the copula method selects one observationally equivalent combination of the supply and demand functions, and the resulting estimates of the parameter of interest are inconsistent.

## 6.2 Example 2: Oligopoly Pricing with Logit Demand

We next consider two imperfect competition pricing examples, both of which are standard environments in industrial organization (see, e.g., Berry, 1994; Nevo, 2001; Dubé and Misra, 2023). The first one is a monopoly pricing example featuring heterogeneity across two unobservables: cost and demand shocks. The second one is a duopoly pricing game, in which we shut down cost heterogeneity entirely to isolate the effect of multiple demand shocks on pricing behavior.

**Model and Econometrics.** In the monopoly case, consumers choose between purchasing the monopolist's product or an outside option, with the purchase probability given by the logit form:

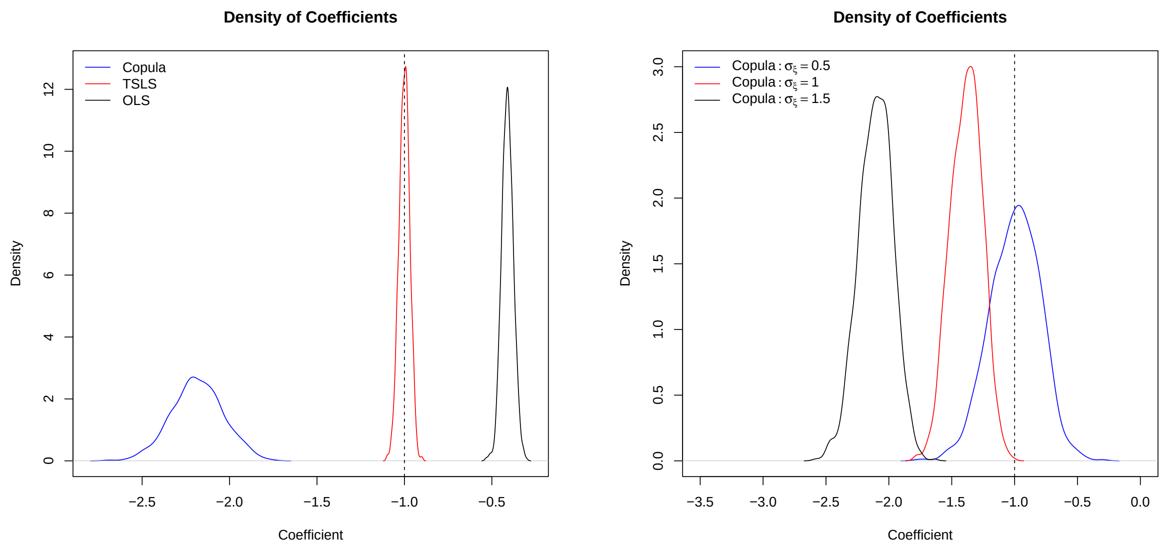
$$s(p_t, \xi_t) = \frac{\exp\{\alpha p_t + \xi_t\}}{1 + \exp\{\alpha p_t + \xi_t\}}, \tag{15}$$

where $p_t$ is price and $\xi_t \sim \mathcal{N}(0, 0.5^2)$ is an unobserved demand shock. The firm observes $\xi_t$ and a marginal cost $c_t \sim \log\mathcal{N}(-0.5, 0.4^2)$, setting price to maximize profits. The first-order condition yields:

$$p_t = c_t - \frac{1}{\alpha\,(1 - s(p_t, \xi_t))}. \tag{16}$$

If marginal cost is constant, $p_t$ is a strictly monotone, non-linear transformation of $\xi_t$, satisfying the conditions of Theorem 1 exactly. However, if $c_t$ varies, this monotonicity

**Figure 2:** Estimates of the demand coefficient $\alpha$ for Example 2




A) Monopoly pricing B) Oligopoly pricing

Notes: The dotted line is positioned at $x = -1$ (demand slope). Panel A has an ICON score of less than 6 (Qian et al., 2026).

breaks. We invert the demand system (Berry, 1994) to estimate the linear parameter $\alpha = -1$, comparing the performance of OLS, the copula correction method, and two-stage least squares.

In the duopoly pricing example, firms $j \in \{1, 2\}$ sell one differentiated product each, and consumers choose among these products and an outside option using choice probabilities analogous to the ones in Equation 15. Solving the Bertrand-Nash price-competition game yields the equilibrium pricing equation:

$$p_{jt} = c_{jt} - \frac{1}{\alpha\left(1 - s_j(\mathbf{p}_t, \boldsymbol{\xi}_t)\right)}. \tag{17}$$

Because the market share $s_j$ depends on both products' prices and shocks, firm $j$'s price is a function of both its own unobserved shock ($\xi_{jt}$) and its rival's ($\xi_{kt}$). We set $c_{jt} = 0$ and $\alpha = -1$, estimating the model across varying standard deviations for the demand shocks ($\xi_{jt} \sim \log(N(-0.5, \sigma_\xi^2))$, with $\sigma_\xi \in \{0.5, 1, 1.5\}$).

**Results.** Figure 2.A shows that the copula method fails in the monopoly pricing case, which follows from a violation of the necessary conditions in Theorem 1. The figure further shows that the copula method estimates are, on average, farther from the true parameter $\alpha = -1$ than OLS estimates, while two-stage least squares yields estimates centered around the truth.[10]

Figure 2.B shows that the copula method only succeeds in the duopoly pricing setting when shock variance is low ($\sigma_\xi = 0.5$), as the rival's shock has a negligible second-order effect on $p_{jt}$. At larger variances, the presence of the rival's shock creates severe omitted variable bias. Consistent with Proposition 3, a correction term built solely from the firm's own price cannot absorb the multidimensional unobservables, pulling estimates progressively further from the truth. The copula method performs adequately when $\sigma_\xi = 0.5$, but not for higher values of $\sigma_\xi$. The researcher, however, does not observe $\sigma_\xi$, and therefore cannot assess the severity of the endogeneity bias in any given application.

### 6.3 Example 3: Advertising Effects

Our last example applies the copula method to the estimation of the elasticity of sales with respect to advertising, a classic question in economics and marketing. Qian et al. (2026) find 28 articles that apply the copula method to the estimation of the impact of promotion (which includes advertising). As is the case with pricing, the marketing literature recognizes the econometric concern over endogeneity, but generally does not model the endogenous determination of advertising. We do so here, adopting an approach introduced by Dorfman and Steiner (1954) and developed into a complete dynamic model by Nerlove and Arrow (1962). Their setting posits a constant-elasticity $\eta$ demand function,

$$q_t = \xi_t A_t^\theta p_t^{-\eta}, \tag{18}$$

where (as before) $p_t$ is price and $\xi_t$ is an unobserved demand shock. The parameter to be estimated is $\theta$, the elasticity of demand with respect to advertising.

**Model and Econometrics.** A monopolist chooses price ($p_t$) and the amount of advertising ($A_t$) to maximize profits:

$$\pi_t = (p_t - c_t)q_t - d_t A_t = (p_t - c_t)\,\xi_t A_t^\theta p_t^{-\eta} - d_t A_t, \tag{19}$$

[10]The copula method and OLS estimates suggest an ICON value less than six (Qian et al., 2026).

where $c_t$ are marginal costs of production and $d_t$ is the cost of a unit of advertising (minute of air time, for example). There are three endogenous variables determined in equilibrium: quantity ($q_t$), price ($p_t$), and advertising ($A_t$). Maximizing with respect to $p_t$ and $A_t$ yields a constant proportional markup for price, $p_t^* = \frac{\eta}{\eta-1}c_t$, and the following optimality condition for advertising:

$$\frac{\partial \pi_t}{\partial A_t} = \theta(p_t - c_t)\xi_t A_t^{\theta-1} p_t^{-\eta} = d_t. \tag{20}$$

Substituting the optimal price into this condition gives the equilibrium advertising choice:

$$A_t^* = \kappa \left( \frac{\xi_t c_t^{1-\eta}}{d_t} \right)^{\frac{1}{1-\theta}}, \tag{21}$$

where $\kappa \equiv \left(\frac{\theta}{\eta-1}\right)^{1/(1-\theta)} \left(\frac{\eta}{\eta-1}\right)^{-\eta/(1-\theta)}$. With $\theta < 1$ and $\eta > 1$, optimal advertising is increasing in demand shocks and decreasing in cost shocks. The OLS estimate will be biased upward: advertising covaries positively with the composite error through both channels — it increases with the demand shock, and it decreases with marginal cost, which enters the composite error negatively. Fortunately, if $d_t$ is observable, it is a natural instrumental variable, being both relevant and excludable in many contexts.[11]

Taking logs of equation (18) yields the linear estimation equation $\ln q_t = \theta \ln A_t - \eta \ln p_t + \ln \xi_t$. This raises the question of how to handle two endogenous variables on the right hand side. Dall'Olio and Vakratsas (2023) include separate copula terms for ad spending and price in the estimation. To keep the focus on a single endogenous variable, we exploit the constant-markup implication of the model. Theoretically, this neutralizes price endogeneity if costs are observed. We focus on identifying the advertising elasticity $\theta$ for situations where neither price nor cost data are available. We do this by absorbing costs into the error term:

$$\ln q_t = \theta \ln A_t + \upsilon_t, \tag{22}$$

where the composite error is $\upsilon_t = \ln \xi_t - \eta \ln c_t$. Because two structural shocks ($\xi_t$ and $c_t$) combine into this single composite error $\upsilon_t$, the dimensionality of the unobservables in the estimating equation falls from two to one, which in principle makes the environment more favorable to the copula method.

Sethuraman et al. (2011) conduct a meta-analysis of advertising elasticities and report a mean for the long-run elasticity of 0.24. Own price elasticities in the IO literature that

[11]For example, heterogeneity in $d_t$ could come from changes in the media competition landscape.

we know of typically range between 3 and 5. Our simulation of the advertising problem therefore assumes $\theta = 0.24$ and $\eta = 4$. As before, the demand shock, $\ln \xi_t$, is drawn from the standard normal distribution. To ensure that $\ln A_t$ will not be too collinear with the copula term, we choose distributions for $\ln c_t$ and $\ln d_t$ that are highly skewed. Both are assumed to be reflected exponential shocks. That is $\ln j = \sigma_j (1 - \varepsilon_j)$ with $\varepsilon_j \sim \text{Exp}(1)$ for $j \in \{c, d\}$. These distributions can be motivated from the underlying assumption that productivity in manufacturing and advertising are both Pareto distributed. While these assumptions may seem contrived, we chose them to mitigate the collinearity that would prevent the use of the copula correction. In our Monte Carlo simulations, we vary the standard deviation of log marginal costs while holding the standard deviation of advertising costs at a high enough level ($\sigma_d = 2$) to keep the ICON score below five for all considered $\sigma_c$. This is safely below the threshold of six recommended in Qian et al. (2026).

**Table 1:** Copula, IV, and OLS estimates of the advertising elasticity (true value: 0.24)

| | Std. dev. of ln costs ($\sigma_c$) | | | |
|---|---|---|---|---|
| Method | 0 | 0.25 | 0.5 | 1.0 |
| IV | 0.240 | 0.240 | 0.240 | 0.239 |
| | (0.006) | (0.008) | (0.012) | (0.022) |
| OLS | 0.392 | 0.479 | 0.659 | 0.945 |
| | (0.006) | (0.008) | (0.012) | (0.011) |
| Copula | -0.042 | -0.031 | 0.351 | 1.105 |
| | (0.025) | (0.033) | (0.055) | (0.042) |
| ICON | 4.120 | 3.930 | 4.660 | 3.780 |

Mean over 1000 simulations (5000 firms each)
Standard deviations of simulations shown in ().
$\ln c = \sigma_c (1 - \varepsilon_c), \qquad \varepsilon_c \sim \text{Exp}(1)$
Demand shocks distributed $\ln \xi_t \sim N(0, 1)$.
$\ln d = \sigma_d (1 - \varepsilon_d), \qquad \varepsilon_d \sim \text{Exp}(1), \sigma_d = 2$.
Other parameters: $\theta = 0.24$, $\eta = 4$.

**Results.** As Table 1 shows, the copula correction fails to recover the true parameter ($\theta = 0.24$). While IV estimation is centered on the true value of $\theta = 0.24$, the OLS estimates are much higher, reflecting the systematic bias discussed above. When cost variation is added in and increased, the bias of OLS increases because then it also builds in the additional source of endogeneity that low-cost firms who expect to sell a lot also choose higher advertising. Substituting the composite error $\upsilon_t$ into the firm's log-linearized optimal advertising rule yields $\ln A_t = \kappa + \frac{1}{1-\theta}(\upsilon_t - \ln d_t) + \frac{1}{1-\theta} \ln c_t$. The endogenous regressor remains driven by both the composite error and unobserved marginal cost, directly violating the single-unobservable assumption. In the first two settings (with $\sigma_c$ of 0 or 0.25), the copula correction is actually biased downwards (even obtaining the incorrect sign, although not significantly so). With more cost heterogeneity in columns three and four, the copula estimator is biased upwards. The copula bias is similar to that of OLS when $\sigma_c = 1$.

The standard deviations (shown in parentheses below the means) of the copula estimates are four to five times higher than the OLS estimates. ICON, which reports this ratio, is a measure of efficiency, rather than bias. It does not appear to be informative about the relative bias between OLS and Copula. The third column ($\sigma_c = 0.5$), is the sole case in which the Copula estimate is closer on average to the true value (0.24, also estimated by IV) than OLS. However, this setting obtains the worst ICON value. As we have seen in the previous sections, passing the ICON test does not imply that the copula estimator yields unbiased estimates. Here, the presence of multiple sources of heterogeneity in the endogenous variable (demand shocks, production cost shocks, and advertising cost shocks) violates the conditions that the copula correction requires. Since we think it likely that real world advertising decisions reflect all these causal factors, it is not encouraging for the application of the copula method to advertising.

# 7 Practical Considerations

We note that the results in Sections 4 and 5 concern identification: they hold with knowledge of the population distribution of the data. Even in the knife-edge case of Theorem 1, in which the method is identified, practical complications arise in estimation. This section briefly discusses three practical considerations; the first two have been examined elsewhere.

## 7.1 Choice of Cumulative Distribution Function Estimator

The correction term requires the marginal distribution $F_P$, which must be estimated. Replacing $F_P$ with an estimate $\hat{F}_P$ breaks the exact equality $\Phi^{-1}(F_P(p_t)) = \xi_t$ in any finite sample, reintroducing correlation between the residual and the endogenous regressor (Section 3.1). The resulting bias vanishes asymptotically under the conditions of Theorem 1, but the finite-sample behavior of the estimator depends on the choice of CDF estimator. See Qian et al. (2026) for a broader discussion about CDF estimation.

### 7.2 Covariates and Conditional Distributions

In applications with covariates, the construction in Theorem 1 requires the conditional distribution $F_{P|X}$ rather than the marginal distribution $F_P$: the argument of Step 1 of the proof applies conditional on the exogenous variables, and using the unconditional CDF when prices systematically vary with covariates introduces a misspecified correction term. Estimating $F_{P|X}$ is a more demanding exercise than estimating a univariate CDF. See Hu et al. (2025) for more discussion on the issue of computing the copula correction term in the presence of covariates.

### 7.3 Functional-Form Identification

Identification in the knife-edge case is identification by functional form. As in a Heckman-style selection correction without an exclusion restriction (Heckman, 1979), the parameters are identified by the non-linearity of the correction term relative to the other regressors, and the observations that carry the identifying information are those in the regions where the non-linearity is pronounced—typically the tails of the price distribution. Estimates are accordingly sensitive to outliers and to the behavior of $\hat{F}_P$ in regions where data are sparse.

## 8 Concluding Remarks

We characterize when the Gaussian copula correction of Park and Gupta (2012) identifies structural parameters and when it does not. We find that the correction identifies the structural parameters when three conditions hold jointly: a single unobservable drives the endogenous variable, the researcher knows the marginal distribution of the endogenous component of the structural error, and the structural mapping from the unobservable to the endogenous variable is strictly non-linear, so that the endogenous variable and this error component are not identically distributed up to an affine transformation.

We show that the correction does not identify the structural parameters in simultaneous systems in which multiple equations with multiple unobservables determine the endogenous variables. In those settings—which include supply–demand systems and monopoly and oligopoly pricing, the canonical environments of demand estimation—the copula correction term fails to recover the endogenous component of the error term, and the structural parameters are observationally equivalent to a continuum of alternatives. The core issue is one of dimensionality: no function of $J$ prices can recover $k \times J$ unobservables with $k \geq 2$.

From a practical perspective, our findings have a direct implication. The settings in which applied researchers most often reach for the copula correction—demand estimation with prices set by optimizing firms facing both demand and cost shocks—are settings in which the method's identifying assumptions are inconsistent with the economic structure that generates the data. Our simulations show that the resulting bias is quantitatively large, and that the copula correction may yield estimates farther from the truth than OLS.

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