Innovation and Competition: The Role of the Product Market*

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Abstract

We study how competition impacts innovation (and welfare) when firms compete both in the product market and in innovation development. This relationship is complex and may lead to scenarios in which a lessening of competition increases R&D and consumer welfare in the long run. We provide conditions for when competition increases or decreases industry innovation and welfare. These conditions are based on properties of the product market payoffs. Implications for applied work and policy are discussed.

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1 Introduction

Merger policy is based on the premise that a lessening of competition is likely to hurt consumers. This view has guided the analysis of mergers in innovative industries on both sides of the Atlantic despite a lack of consensus on how competition impacts innovation outcomes.[1] For example, Aghion *et al.* (2005) empirically find a non-monotonic relationship between competition and patenting, which raises the possibility that a lessening of competition may benefit consumers through enhanced innovation. To inform this debate, we seek to provide applied researchers with conditions for when to expect monotonic or non-monotonic relationships between competition and innovation.

We analyze how competition affects firms' incentives to innovate and consumer welfare, focusing on the role played by the product market. To this end, we propose a *dynamic* model of an innovative industry that accommodates arbitrary product market games (e.g., quantity or price competition with either homogeneous or differentiated products), and study how the product market game being played by the firms shapes the relationship between competition and innovation. The motivation behind examining the role played by the product market stems from the observation that firms invest in R&D because they wish to gain a product market advantage (e.g., a greater product quality or a lower marginal cost). Because competition impacts product market payoffs, competition impacts the incentives to invest in R&D through the product market.

In concrete terms, we develop a sequential extension to the classic patent-race models (Loury 1979, Lee and Wilde 1980, and Reinganum 1982), where we consider two types of firms. *Large* firms competing in developing innovations and in the product market; and research *labs* only competing in developing innovations. A market has $n+1$ large firms competing in the product market, and $n+m+1$ firms competing in developing innovations, where m is the number of research labs. The distinction between large firms and labs captures the fact that firms are asymmetric in both size and scope in many innovative industries (e.g., pharmaceutical

[1] See, for instance, the complaint filed by the Federal Trade Commission (FTC) concerning the merger between Pfizer Corporation and Wyeth Corporation, as well as the complaints filed by the Department of Justice (DOJ) concerning the merger between Regal Beloit Corporation and A.O. Smith Corporation and the merger between The Manitowoc Company, Inc. and Enodis plc. A similar argument was provided by the European Commission (EC) in its investigations of Qualcomm's proposed acquisition of NXP, Bayer's proposed acquisition of Monsanto, and the proposed merger between Dow and DuPont. In fact, the Dow and DuPont merger was cleared by the European Commission subject to a divestiture of DuPont's R&D organization.

industry).

Through successful innovation, a large firm becomes the market leader, replacing the previous leader. When a research lab successfully innovates, it auctions the innovation to a large firm, which results in a new industry leader. Being the leader provides a firm with an advantage in the product market—for instance, due to a cost or quality advantage—which creates a positive *profit gap* between the leader and the followers. This profit gap captures the profit market advantage of the market leader.

In the model, competition affects innovation through two channels. First, holding product market profits equal, a reduction in the number of firms performing R&D reduces the pace of innovation in the industry (Reinganum, 1985). Most of the patent race literature has focused on this first mechanism. Secondly, because competition has a direct effect on the product market payoffs and, consequently, the profit gap that exists between the leader and the followers, competition in the product market affects the incentives to innovate. Depending on the specifics of the product market game, a lessening of product market competition may increase or decrease the profit gap between leaders and followers. This creates a potentially countervailing effect on the incentives to innovate, which may generate a monotonic-increasing or non-monotonic relationship (e.g., inverted-U or N shaped) between innovation outcomes and the number of large firms.

Although the relationship between competition and innovation may in principle take various shapes, the product market game being played by the firms puts restrictions on this relationship. Product market games can be categorized according to the properties of an equilibrium object: the profit gap between the leader and followers.

We show that when the profit gap between the leader and followers is *weakly increasing* in the number of large firms, the industry's innovation rate is always increasing in the number of large firms. Because competition in the product market also (weakly) decreases equilibrium prices, the number of large firms unequivocally increases the discounted expected consumer surplus in this case. Product market games that feature a weakly increasing profit gap include some parameterizations of price and quantity competition games with homogeneous goods.

We also show that a profit gap between the leader and followers that is *decreasing* in the number of large firms is necessary but not sufficient for the industry's innovation rate to decrease in the number of large firms. When the number of research labs is sufficiently large, however, a profit gap that is decreasing in the number of large firms is sufficient for the industry's innovation rate to decrease in the number of large firms. Some parameterizations of quantity competition games with homogeneous goods and price competition games with differentiated products are examples that feature a decreasing profit gap.

Because competition in the product market decreases equilibrium prices, a negative relationship between competition and innovation is not one-to-one with a negative relationship between competition and consumer welfare. However, we show that when the number of research labs is sufficiently large, a profit gap that is decreasing in the number of large firms suffices for the discounted expected consumer surplus to decrease in the number of large firms. That is, there are scenarios in which a lessening of competition may increase consumer welfare in the long run. In these scenarios, the increased arrival rate of innovations more than compensates for the welfare loss that results from static price effects.

We finish by analyzing the impact of product market competition on total welfare. While consumers always benefit from R&D investments in our model, R&D investments are wasteful from the perspective of firms as a whole because these are purely driven by business stealing. This implies that one extra dollar spent in R&D only benefits society if the consumer benefits exceed one dollar. Because the number of large firms impacts both prices and R&D investments, a change in competition may for example benefit consumers through enhanced innovation while harm society because of excessive R&D investments. Although the relationships between competition and consumer and total welfare may not always be aligned, we provide sufficient conditions for them to align. These sufficient conditions crucially depend on whether the profit gap between the leader and followers increases or decreases in the number of large firms.

These results have three broader implications. First, these results are constructive in that they isolate a specific property of the product market payoffs that is key for understanding the relationship between innovation and competition. Second, our analysis calls for the use of flexible demand systems when using an empirical model to measure the impact of competition on market outcomes in innovative industries. A lack of model flexibility may restrict the product market payoffs in ways that prevent the model from showing the true relationship between competition and innovation in the data.

Lastly, our results have implications for competition policy along two dimensions. First, we provide conditions that identify scenarios where a lessening of competition may harm consumers. These conditions are easy to check, as they only depend on properties of the product market game. Second, the finding that the relationships between competition and consumer and total welfare may not align suggests that different policy recommendations may arise depending on the specific criterion used to evaluate horizontal mergers (i.e., consumer or total welfare). Our results provide sufficient conditions for when different criteria used to evaluate horizontal mergers are aligned.

The rest of the paper is organized as follows. Section 2 introduces the model and characterizes the equilibrium. Section 3 analyzes how market structure affects innovation and welfare outcomes. Section 4 provides numerical examples to illustrate the results. Lastly, Section 5 concludes.

1.1 Literature Review

The question of how competition affects the incentives to innovate stems from the work of Schumpeter (1942).[2] The literature has taken two approaches to modeling the relationship between R&D investments and its returns. The first branch assumes that R&D investments deliver deterministic returns. The second branch—the patent race literature—assumes a stochastic link in which greater investments lead to greater innovation rates.

In a deterministic-R&D model, Dasgupta and Stiglitz (1980a) (henceforth, DS) study the role of product market competition in a scenario in which symmetric firms compete á la Cournot and in developing process innovations. Under an isoelastic demand assumption, the authors show that an increase in the number of firms decreases each firms' investments, but increases aggregate investments. More recently, Vives (2008) generalizes these findings by allowing for a broader set of demand functions and price competition games. Ishida *et al.* (2011) shows that the assumption of symmetric firms is critical for the results in DS: in quantity competition models with high- and low-costs firms, an increase in the number of high-cost firms only leads to DS's result among high-cost firms, as low-cost firms experience enhanced incentives to innovate. In price competition models, Motta and Tarantino (2017) identify conditions under which DS's result holds when the reduction of competition is due to a merger. In contrast to these papers, we show that once dynamics are incorporated, the relationship between competition

[2]See Gilbert (2006) and Cohen (2010) for surveys of the literature.

and innovation is richer than previously described. We illustrate this in Section 4, where we present an example satisfying DS's original assumptions about the product market game—Cournot competition with isoelastic demand—in which a lessening of competition may lead to enhanced or reduced innovation incentives both at the individual and aggregate level.

Early work in the patent race literature often omitted the role of product market competition and dynamic considerations (Loury 1979, Lee and Wilde 1980, and Reinganum 1982).[3] In a single-innovation model, Dasgupta and Stiglitz (1980b) modeled payoffs as the result of product market competition, but did not study how competition affects innovation outcomes. Reinganum (1985) incorporated dynamics by studying a sequence of patent races where firms compete through a ladder of innovations. She finds results analogous to DS in a context where product market payoffs are unaffected by the number of competitors. Aghion *et al.* (2001), Aghion *et al.* (2005), and other follow-up papers have examined the impact of product market competition on innovation decisions in duopolistic markets. In these models, the duopolists compete in prices, and competition is captured by the degree of substitution between the products sold by the firms. A key observation in these papers is that innovation is driven by the "escape competition" effect, i.e., the difference between the payoffs before and after the introduction of an innovation. We build upon these ideas by extending the model to an arbitrary number of firms and allowing for general product market games. We directly link the escape of competition effect (and, thus, R&D outcomes) with product market outcomes. A key finding in our paper is that the relation between competition and innovation is determined by a combination of how firms compete (quantity or prices) and the shape of the demand function. This finding has consequences for empirical work, as some parametric choices may lead to empirical models that are not flexible enough to capture the effects of competition on innovation.

Our analysis is built upon a standard dynamic model of innovation. Versions of the model have been used by Aghion and Howitt (1992) to study endogenous growth; Segal and Whinston (2007) to study the impact of antitrust regulation on innovation outcomes; Acemoglu and Akcigit (2012) to study an IP policy contin-

[3]Although the patent race literature has a dynamic dimension—the expected arrival time of innovations—we use the term dynamic to incorporate the intertemporal tradeoffs that arise when firms compete in developing a sequence of innovations and in the product market. For example, the tradeoff that arises when a lessening of competition increases prices in the short run but enhances the rate at which innovations reach the market is an intertemporal tradeoff that is absent in the early patent race literature.

gent on the technology gap among firms; Denicolò and Zanchettin (2012) to study leadership cycles; Acemoglu *et al.* (2013) to study productivity growth and firm reallocation; and by Parra (2019) to study the dynamics of the Arrow's replacement effect and its impact on patent design.

Our paper also relates to the horizontal merger literature.[4] Several authors have discussed at a conceptual level how innovation considerations should be incorporated into merger analysis (see, for instance, Gilbert and Sunshine 1995, Evans and Schmalansee 2002, Katz and Shelanski 2005, 2007), and a number of recent papers have explored this issue empirically or using computational methods.[5] Although we do not model merger decisions (or merger-specific synergies), we contribute to this literature by providing analytic results that clarify the role played by market concentration on firms' investment decisions.

2 A Model of Sequential Innovations with Product Market Competition

Consider a continuous-time infinitely lived industry where $n+m+1$ firms compete in developing new innovations (or products). Among these, $n+1$ firms are *large* in the sense that they also compete in the product market selling final products. The remaining m firms auction their innovations to the large firms; we call the latter set of firms *research labs.* This model thus considers $n+1$ product market competitors and $n+m+1$ firms competing in developing innovations.

Competition in the product market is characterized by one technology leader and $n > 0$ symmetric followers (or competitors). The market leader obtains a profit flow $\pi_n^l > 0$, whereas each follower obtains a profit flow $\pi_n^f \in [0, \pi_n^l)$. We interpret these profit flows as the equilibrium payoffs that result from an arbitrary product market game. With respect to innovation competition, the model allows for different types of innovations: firms may compete in developing process inno-

[4]See, for example, Williamson (1968), Farrell and Shapiro (1990), Gowrisankaran (1999), Nocke and Whinston (2010, 2013), Federico *et al.* (2017).

[5]Ornaghi (2009) studies mergers in the pharmaceutical industry and their impact in R&D. Igami and Uetake (2015) studies the relation between mergers and innovation in the hard-drive industry and Entezarkheir and Moshiri (2015) performs a cross-industry analysis. Mermelstein *et al.* (2015) and Hollenbeck (2015) use computational methods to study optimal merger policy in a dynamic oligopoly model with endogenous capital and R&D investments, while Federico *et al.* (2018) simulate the impact of a horizontal merger on consumer welfare and innovation using a static model.

vations, quality improvements, or products that leave previous vintages obsolete. For tractability purposes, we assume that the market leader is always one step ahead of the followers in terms of the technology to which they have access.[6] We relax this assumption in the Online Appendix, where we allow the leader to invest in increasing its technological lead relative to the followers.

We make two assumptions about the equilibrium profit flows. First, we assume that both π_n^l and π_n^f are weakly decreasing in the number of product market competitors in the industry (i.e., large firms), $n+1$, capturing that more intense product market competition decreases the profits of all firms. Second, and for the purpose of reducing the dimensionality of the state space and making analytic results feasible, we assume that the profit flows are stationary in the number of innovations. Our results, however, do not depend on this stationarity assumption. Sections 3 and 4 provide examples where all the assumptions of the model are satisfied.

Research labs do not compete in the product market and their only source of profits is the revenue they derive from selling their innovations to large firms. We assume that research labs sell their innovations using a second-price auction. In case of a tie, we assume that the innovation is randomly assigned to one of the tying followers.[7] Other than helping us capture market structure in innovative industries in a more realistic way, incorporating research labs into the model will be helpful because it will allow us to vary the number of firms competing in developing innovations $(n+m+1)$ without changing the number of firms competing in the product market $(n+1)$. This will be useful in establishing some of our results.

At each instant in time, every follower and research lab invests in R&D in order to achieve an innovation. Firm i chooses a Poisson innovation rate x_i at a cost of $c(x_i)$. We assume that $c(x_i)$ is strictly increasing, twice differentiable, strictly convex (i.e., $c''(x) > 0$ for all $x \geq 0$), and satisfies $c'(0) = 0$. The assumption that large firms and labs are equally productive along the R&D dimension is for notational ease. Introducing asymmetries does not impact our results in a significant way. We also assume that the Poisson processes are independent among firms, generating a stochastic process that is memoryless. All firms discount their future

[6] More precisely, this common assumption in the literature can be distilled as the conjunction of two independent assumptions about the nature of patent protection: a) a patent makes full disclosure of the patented technology, which allows followers to build upon the latest technology, leap-frogging the leader once they achieve an innovation; b) the legal cost of enforcing older patents more than exceeds the benefits of enforcing the patent.

[7] This assumption simplifies exposition and does not affect the results of the paper.

payoffs at a rate of $r > 0$.

We focus on symmetric and stationary Markov perfect equilibria by using a continuous-time dynamic programming approach. Our assumptions guarantee the concavity of the value functions, implying equilibrium uniqueness.

Let $V_{n,m}$ represent the value of being the market leader, $W_{n,m}$ the value of being a follower, and $L_{n,m}$ the value of being a research lab when there are n followers and m labs in the industry. At time t, we can write the value functions of the different types of firms as follows:

$$rV_{n,m} = \pi_n^l - \lambda_{n,m}(V_{n,m} - W_{n,m}), \tag{1}$$

$$rW_{n,m} = \max_{x_i} \ \pi_n^f + x_i(V_{n,m} - W_{n,m}) - c(x_i), \tag{2}$$

$$rL_{n,m} = \max_{y_i} \ y_i(V_{n,m} - W_{n,m}) - c(y_i), \tag{3}$$

where $\lambda_{n,m} = \sum_i^n x_i + \sum_j^m y_j$ captures the rate at which innovations reach the market (i.e., the industry-wide *pace* or *speed* of innovation).[8] In words, the flow value of being the market leader at any instant of time, $rV_{n,m}$, is equal to the profit flow obtained at that instant plus the expected loss if an innovation occurs, $\lambda_{n,m}(W_{n,m} - V_{n,m})$, where $\lambda_{n,m}$ is the rate at which some firm successfully innovates. The instantaneous value of being a follower, $rW_{n,m}$, is equal to the profit flow plus the expected incremental value of becoming the leader, $x_i(V_{n,m} - W_{n,m})$—where x_i is the rate at which follower i successfully innovates—minus the flow cost of R&D, $c(x_i)$. Finally, the flow value of being a research lab is equal to the expected payoff of successfully innovating and selling an innovation, $y_i(V_{n,m} - W_{n,m})$—where y_i is the rate at which lab i successfully innovates—minus the flow cost of R&D, $c(y_i)$. Note that since all large firms are symmetric, large firms value an innovation in $V_{n,m} - W_{n,m}$. These valuations, in conjunction with the auction format, imply that labs sell their innovations at price $V_{n,m} - W_{n,m}$ in equilibrium.[9]

In the context of this model, the infinitely long patent protection and the assumption that a new innovation completely replaces the old technology implies that the incumbent has no incentives to perform R&D. That is, the leader's lack of R&D is an implication of our modeling choices rather than an assumption; see Parra (2019) for a formal proof. In the Online Appendix, we extend the model to

[8]See the Appendix for a full derivation of the value functions.

[9]Since the winning bidder of an auction held by a lab earns zero surplus, we do not include auction payoffs in the value functions of the leader and followers.

allow for the leader to increase the quality of its innovation by investing in R&D. We further discuss this extension below.

Maximizing value functions (2) and (3), and imposing symmetry among followers and research labs, we obtain $x_i = y_i = x^*_{n,m}$, where

$$c'(x^*_{n,m}) = V_{n,m} - W_{n,m} \tag{4}$$

or $x^*_{n,m} = 0$ if $c(0) > V_{n,m} - W_{n,m}$; with the subindices n and m capturing how market structure affects R&D decisions. Equation (4) tells us that, at every instant of time, the followers and research labs invest until the marginal cost of increasing their arrival rate is equal to the incremental rent of achieving an innovation. The incremental rent of achieving an innovation relates to the "escape competition" effect in Aghion *et al.* (2001), as $V_{n,m} - W_{n,m}$ represents the benefits of escaping competition through an innovation.

Strict convexity implies that condition (4) can be inverted so that $x^*_{n,m} = f(V_{n,m} - W_{n,m})$, where $f(z)$ is a strictly increasing function of z.[10] By replacing $x^*_{n,m}$ into equations (2) and (3), we can solve the game and prove the following proposition.

Proposition 1 (Market equilibrium)**.** *There is a unique symmetric equilibrium, which is determined by the solution of the system of equaetions (1–4).*

It can be easily verified that the payoffs in this model possess the expected comparative statics for given values of n and m. For instance, the value functions increase with larger profit flows or a lower interest rate (all else equal).

2.1 Discussion of Modeling Assumptions

We further discuss two modeling assumptions in this subsection. As mentioned above, market leaders do not invest in R&D due to Arrow's replacement effect. That is, because an innovation replaces the current technology with one of equal value, the leader does not have an incentive to build upon its own technology. In the Online Appendix, we develop an extension of the model where the leader can invest to improve its innovation, and thus increase its technology lead relative to the followers. We show that the main findings presented in the next section still hold in the environment with a leader investing in R&D.

[10] This function is further characterized in Lemma 1 in the Appendix.

A second assumption in our model is that the number of large firms and research labs is exogenous. That is, we are silent about the determinants of market structure (e.g., entry costs) and factors that could trigger changes in the number of competitors (e.g., mergers). Because the main purpose of our analysis is to understand how competition in both the product market and innovation impact innovation and welfare—regardless of what factors could explain a change in competition—we chose not to endogenize market structure in any way. For example, market structure could be endogenized by assuming that firms pay a sunk cost upon entry and entry occurs while profitable (see, for instance, Chen *et al.* 2018). In this case, our results below on how an exogenous change in the number of competitors impacts welfare and innovation would need to be reinterpreted as results on the impact of entry costs on welfare and innovation.

3 Market Structure and Performance

We next study how market structure affects R&D outcomes and, more generally, welfare. Market structure affects dynamic incentives to invest in R&D through two channels: *product market competition* and *innovation competition.* We explore how these two forms of competition interact and determine market outcomes.

3.1 Pace of Innovation

We begin our analysis by considering how an *isolated* change in innovation competition or an isolated change in product market competition affects innovation outcomes. Although a change in the number of large firms—i.e., firms competing in innovation development and in the product market—affects both forms of competition simultaneously, this exercise gives us a first approach to understanding how each form of competition affects R&D outcomes. The following object is key for our analysis.

Definition. The *profit gap*, $\Delta\pi_n$, is the difference between the equilibrium profit flow of the leader and the equilibrium profit flow of a follower; i.e., $\Delta\pi_n = \pi_n^l - \pi_n^f$.

The profit gap measures the (static) product market benefit of being the market leader. While most models of product market competition predict that both π_n^l and π_n^f are weakly decreasing in n; the profit gap can either increase or decrease with n even when both π_n^l and π_n^f are weakly decreasing in n (see examples in Table 1).

Proposition 2 (Product market and innovation competition)**.** *Competition affects innovation outcomes through two channels:*

i) Product market competition: *Fixing the number of firms, an increase in the profit gap between the leader and a follower, $\Delta\pi_n$, increases each firm's R&D investment, $x^*_{n,m}$, and the pace of innovation in the industry, $\lambda_{n,m}$.*

ii) Innovation competition: *A decrease in the number of research labs, m, decreases the overall pace of innovation in the industry, $\lambda_{n,m}$, but increases each firm's R&D investment, $x^*_{n,m}$.*

Firms' incentives to invest in R&D are driven by the incremental rent obtained from an innovation (see equation (4)). Proposition 2 tells us that the incremental rent is increasing in the profit gap between the leader and the followers, and a greater profit gap increases the pace of innovation. This result implies that because product market concentration changes product market payoffs—and, consequently, the profit gap—product market concentration has an impact on the incentives to invest in R&D.[11] As we shall see later, a specific property of the profit gap determines the shape of the relationship between competition and the pace of innovation.

From Proposition 2 we also learn that innovation competition affects the pace of innovation in two ways. To understand these effects, suppose we decrease the number of research labs by one. Varying the number of labs is convenient because it allows us to abstract away from product market effects, as labs do not compete in the product market. First, the reduction in the number of firms performing R&D has a direct negative effect on the pace of innovation in the industry, $\lambda_{n,m}$ (i.e., fewer firms performing R&D). Second, this reduction in $\lambda_{n,m}$ increases the expected time between innovations, extending the lifespan of a leader and raising the value of being a market leader, $V_{n,m}$. This causes an increase in the incremental rent of an innovation, incentivizing the remaining firms to invest more in R&D. Although

[11]It is through this channel that our analysis differs from the growth through innovation literature (e.g., Aghion *et al.* 2001), which has examined how the intensity of product market competition—captured by the degree of substitution among a fixed number of firms or the degree of collusion between firms—affects innovation. In our analysis, we explicitly study how a change in the number of competitors affects innovation through changes in product market payoffs. Our analysis encompasses substitution effects as well as various forms of competition and types of innovations.

each remaining firm increases its R&D investment, the first effect dominates, and the lessening of innovation competition leads to a decrease in the industry's pace of innovation.[12] A similar result is discussed in Reinganum (1985).

Proposition 2 illustrates how product market competition and innovation competition affect the incentives to innovate in isolation. A change in the number of large firms $(n+1)$, however, affects both forms of competition simultaneously. The interaction between these forms of competition is complex, as these effects may either reinforce or collide with each other. Because of the interaction of these effects, the relationship between the number of large firms $(n+1)$ and innovation may be monotonic or non-monotonic. Figure 1 shows some examples based on parameterizations of the model that we discuss in Section 4.

Although the relationship between competition and innovation may in principle take various shapes, the product market game being played by the firms puts restrictions on this relationship. Product market games can be categorized according to an equilibrium object: the profit gap between the leader and followers.

Definition. The equilibrium profit flows have a *decreasing profit gap* between the leader and a follower when the profit gap $(\Delta\pi_n)$ is decreasing in the number of large firms, $n+1$. Likewise, the equilibrium profit flows have an *increasing profit gap* between the leader and a follower when the profit gap is increasing in the number of large firms, $n+1$.[13]

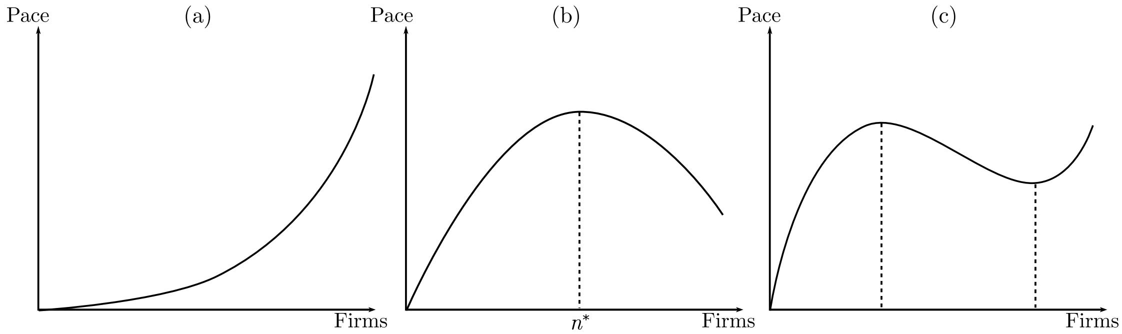
Table 1 shows examples of product market games, and provides information about the shape of the profit gap in each example. For instance, a constant-elasticity demand in a quantity competition game can deliver a profit gap that is increasing or decreasing in the number of firms depending on the value of the elasticity of demand.[14] In what follows, we explore how the shape of the profit gap between the leader and followers puts restrictions on the relationship between competition and innovation.

[12] The net effect of a decrease in the number of research labs on $\lambda_{n,m}$ must be negative, as it was the initial decrease in the pace of innovation that triggered the increase in the incremental rent of an innovation in the first place.

[13] Alternatively, we could have defined firm i's equilibrium profit flow as a function of both the number of large firms and an indicator for whether a firm is the market leader. In this alternative formulation, an increasing (decreasing) profit gap is equivalent to the equilibrium profit flow being supermodular (submodular) in these two variables.

[14] We acknowledge that the assumption of payoffs that are stationary in the number of innovations (see Section 2) is violated for some parameter values in columns Cournot I and II of Table 1. However, we emphasize that our results do not depend on this stationarity assumption. We only make the stationarity assumption for analytic tractability.

FIGURE 1: INDUSTRY'S PACE OF INNOVATION VS. NUMBER OF COMPETITORS IN THE INDUSTRY



Proposition 3 (Competition increases innovation)**.** *A weakly increasing profit gap is sufficient for the pace of innovation to be increasing in the number of large firms.*

Proposition 3 shows a sufficient condition that guarantees an increasing relationship between competition and innovation. The logic behind the result is as follows: if the profit gap between the leader and a follower, $\Delta\pi_n$, increases with the number of large firms, then an increase in the number of large firms increases the incentives to perform R&D (see Proposition 2.i). This product-market effect on innovation is reinforced by a larger number of firms performing R&D (see Proposition 2.ii). An example of a product market game with a weakly increasing profit gap is a model of price competition in a market for homogeneous goods where firms develop process innovations.[15]

Proposition 4 (Competition decreases innovation)**.** *A decreasing profit gap is necessary for the pace of innovation to be decreasing in the number of large firms. If the number of research labs is large enough, a decreasing profit gap is sufficient.*

When the profit gap decreases in the number of firms, a lessening of competition (specifically, a decrease in the number of large firms) creates a tension between the effects of product market competition and innovation competition. On the one hand, the decrease in product market competition increases the profit gap and, consequently, increases the incentives to perform R&D (see Proposition 2.i). On the other hand, a decrease in the number of firms performing R&D has a negative effect on the pace of innovation (see Proposition 2.ii). Although this tension may

[15]In this price competition example, increasing the number of followers (beyond one) does not affect the profit gap, as the market price equals the followers' marginal cost.

Table 1: Product market competition and the slope of the profit gap: examples

	Bertrand	Cournot I	Cournot II	Logit
Differentiation	No	No	No	Yes
Innovation type	Process	Process	Process	Quality ladder
Leader advantage	Marginal cost advantage: $mc_l = \beta mc_f$, $\beta \in (0,1)$			Quality gap: $\kappa > 0$
Demand	$Q = Q(P)$	$Q = a/P^{1/\sigma}$	$Q = a/P^{1/\sigma}$	$s_l = \frac{\exp\{\kappa - p_l\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$ $s_f = \frac{\exp\{-p_f\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$
Restrictions	None	$\frac{(1+\beta)}{(1-\beta)}\frac{\sigma(n-\sigma)}{(n-1)} < 1$	$\frac{(1+\beta)}{(1-\beta)}\frac{\sigma(n-\sigma)}{(n-1)} > 1$	Firm-level horizontal differentiation
Profit gap	Weakly increasing	Increasing	Decreasing	Decreasing

Notes: Subscripts l and f denote leader and follower, respectively. For simplicity, we assume that the horizontal differentiation in the logit model (i.e., the idiosyncratic taste shocks) is at the firm rather than the product level. See Marshall (2015) for an application with a closely related model.

result in an increased pace of innovation (see Section 4 for examples), Proposition 4 shows that in industries in which research labs play an important role in total R&D, a decreasing profit gap between the leader and a follower is sufficient for the pace of innovation to be decreasing in the number of large firms.[16]

The intuition for the sufficiency result in Proposition 4 follows from observing that the R&D incentives of research labs and large firms are aligned (see equation (4)). When product market concentration increases R&D incentives, research labs magnify this effect, as more firms are affected by the enhanced incentives to perform R&D. As shown in Section 4, however, a decrease in the number of large firms may increase the pace of innovation even in the absence of research labs ($m = 0$). That is, the existence of research labs is not necessary for a decreasing relationship between the number of large firms and the pace of innovation, but a large enough number of research labs makes a decreasing profit gap sufficient for the pace of innovation to be decreasing in the number of large firms. We also note here that the auction mechanism used by labs simplifies the analysis, but it is not necessary

[16] The proof that a decreasing profit gap is sufficient for competition to decrease the pace of innovation for a sufficiently large m uses strict convexity of the cost function (i.e., $c''(x) > 0$ for all $x \geq 0$). We note, however, that the result applies for a broader set of cost functions. For instance, the result also applies for all cost functions satisfying $c(x) = x^\gamma/\gamma$ with $\gamma > 1$.

for these results to go through. As long as the labs' incentives are aligned with those of large firms, it follows that labs will magnify the impact of product market competition on R&D outcomes.

In summary, our results show that the product market game played by the firms determines the relationship between competition and innovation. Our results are constructive in that they isolate a specific property of the product market payoffs that is key for understanding this relationship. These findings suggest that model-based research on the impact of competition on innovation should specify product market games that do not ex-ante restrict the relationship between competition and innovation. This is particularly relevant for empirical work, as restrictive empirical models may prevent the analysis from showing the true empirical relationship between competition and innovation in the data.

3.2 Welfare Analysis

We have already provided sufficient conditions for instances when competition increases or decreases the pace of innovation. Evaluating whether an increase in competition is welfare enhancing, however, requires understanding how it affects both the path of prices faced by consumers and the pace of innovation. To this end, we incorporate price effects into the analysis and study the trade-off between the price and innovation effects caused by a change in competition. We first analyze the effect of competition on consumer welfare, and then turn to analyzing the effects on total welfare.

To make statements about the relationship between competition and consumer welfare, we impose further structure to the model.

Assumption 1. *Each innovation increases the consumer-surplus flow by $\delta_n > 0$.*

The term δ_n represents the increment in consumer surplus due to an innovation. If, for instance, firms compete in developing process innovations (i.e., cost-saving technologies), δ_n represents the decrease in cost that is passed on to consumers through lower prices and, consequently, higher consumer surplus. Table 2 provides examples of different demand functions with their respective expressions for the consumer surplus. In all of these examples, a stronger version of Assumption 1 is satisfied: the increment in consumer-surplus flow δ_n is independent of the number of firms competing in the product market, n.

Table 2: Product market competition and consumer surplus: examples

	Bertrand	Cournot	Logit
Differentiation	No	No	Yes
Innovation type	Process	Process	Quality ladder
Leader advantage	Marginal cost advantage: $mc_l = \beta mc_f$, $\beta \in (0,1)$		Quality gap: $\kappa > 0$
Demand	$Q = a/P$ if $P < \bar{P}$		$s_l = \frac{\exp\{\kappa - p_l\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$ $s_f = \frac{\exp\{-p_f\}}{\exp\{\kappa - p_l\} + n\exp\{-p_f\}}$
Consumer-surplus flow (cs_n)	$a \log \bar{P} - a \log p_n$		$\log\left(\exp\{\kappa - p_l\} + n\exp\{-p_f\}\right) + \gamma$
Innovation effect on CS (δ_n)	$-a \log \beta$		κ
Restrictions	None	None	Firm-level horizontal differentiation

Notes: Subscripts l and f denote leader and follower, respectively. The γ parameter in the logit-model consumer surplus is Euler's constant.

Given Assumption 1, the discounted expected consumer surplus, $CS_{n,m}$, which incorporates the dynamic benefits of future innovations, is given by

$$rCS_{n,m} = cs_n + \lambda_{n,m}\delta_n/r, \tag{5}$$

where cs_n is the consumer-surplus flow when there are n product market competitors.[17] Observe that the discounted expected consumer surplus is greater than cs_n and that it is increasing in both the pace of innovation and the magnitude with which each innovation enhances consumer surplus, δ_n. The discounted expected consumer surplus also decreases with the interest rate, as future breakthroughs are discounted at a higher rate.

From equation (5), we can note that competition affects the discounted expected consumer surplus through three mechanisms. First, product market concentration has a direct effect on spot prices, affecting the consumer-surplus flow cs_n. Product market concentration also affects the discounted expected consumer surplus by potentially changing the pass-through of innovations on consumer wel-

[17]See Lemma 2 in the Appendix for the derivation of equation (5).

fare, δ_n. Finally, as discussed in the previous subsection, market concentration has an effect on the pace of innovation, $\lambda_{n,m}$. Because a lessening of competition may increase the pace of innovation at the same time that it increases prices, the relationship between competition and innovation is not one-to-one with the relationship between competition and consumer welfare.

Equation (5) shows that when a lessening of competition increases the market price (i.e., $dcs_n/dn > 0$) and decreases the innovation pass-through on consumer surplus (i.e., $d\delta_n/dn \geq 0$), an increase in the speed of innovation is necessary for a lessening of competition to increase welfare. Based on these observations and our propositions on the relationship between competition and innovation, we can establish the following results on the impact of competition on consumer welfare.

Proposition 5 (Competition and consumer welfare)**.**

i) Suppose competition decreases the market price (i.e., $dcs_n/dn > 0$) and increases the innovation pass-through on consumer surplus (i.e., $d\delta_n/dn \geq 0$). An increasing profit gap between the leader and a follower is sufficient for the discounted expected consumer surplus to increase in the number of large firms.

ii) Suppose competition decreases the market price (i.e., $dcs_n/dn > 0$) and keeps the innovation pass-through on consumer surplus constant (i.e., $d\delta_n/dn = 0$). If the number of research labs is large enough, a decreasing profit gap between the leader and a follower is sufficient for the expected discounted consumer surplus to decrease in the number of large firms.

Proposition 5 first shows that a profit gap that is increasing the number of large firms is sufficient for consumer welfare to be increasing in the number of large firms. This implication is straightforward since, in this case, competition increases the pace of innovation (Proposition 3) at the same that it decreases prices in the short run. The proposition also shows that for a sufficiently large number of labs, and under a restriction on how competition impacts the pass-through of innovations on consumer welfare (i.e., $d\delta_n/dn$), a decreasing profit gap becomes sufficient for consumer welfare to decrease in the number of large firms. The driver of the result is that when product market concentration increases R&D incentives, research labs magnify the effect of competition on the pace of innovation, as more firms are affected by the enhanced incentives. We note that the existence of research labs is not necessary for a decreasing relationship between consumer welfare and the number of large firms (see examples in Section 4), but a large enough number of research labs makes a decreasing profit gap sufficient for consumer welfare to be decreasing in the number of large firms. Also noteworthy is that these sufficient conditions only depend on the number of firms and on properties of the product market payoffs.

We next turn to analyzing the effect of competition on total welfare. Total welfare is given by the sum of the discounted-expected surplus of consumers and firms, $CS_{n,m} + PS_{n,m}$, where $PS_{n,m}$ is the sum of the value of all the firms in the market

$$rPS_{n,m} = r(V_{n,m} + nW_{n,m} + mL_{n,m}) = \pi_n^l + n\pi_n^f - (n+m)c(x^*_{n,m}). \tag{6}$$

The producer surplus flow, $rPS_{n,m}$, consists of the total profits that the large firms earn in the product market minus the aggregate cost of R&D. Because innovation is driven by business stealing—i.e., the stationarity of profit flows in the number of innovations implies that an innovation does not increase the aggregate profits, it only determines the identity of the leader—R&D investments are wasteful from the perspective of firms as a whole. R&D investments, however, benefit consumers, as they increase the rate of arrival of innovations that enhance consumer welfare. Hence, an increase in R&D investments of one dollar increases total welfare as long the benefits to consumers are greater than one dollar. In our analysis of how competition affects total welfare, we must now consider this cost–benefit analysis that was not present when analyzing the effects of competition on consumer welfare.

To better illustrate this tradeoff, consider an increase in the number of large firms in the case of an increasing profit gap. Proposition 3 and Proposition 5 show that consumer welfare and the pace of innovation are increasing in the number of large firms in this case. Firms, on the other hand, face more competition in the product market and collectively spend more in R&D, which lowers the combined value of firms (or producer surplus). These opposing forces suggest that the relationships between competition and consumer and total welfare need not be aligned. In what follows, we provide sufficient conditions for both criteria to align. In the proposition, we make use of the following assumption on how product market competition affects product market surplus (i.e., the gains of trade in the product market at every instant of time)—which rather than an assumption is a property of most product market games.

Assumption 2. *Product market surplus $cs_n + \pi_n^l + n\pi_n^f$ is (weakly) increasing in* *the number of product-market competitors $n + 1$.*

Proposition 6 (Competition and welfare)**.** *Suppose the number of research labs is sufficiently large.*

i) If competition increases the innovation pass-through on consumer surplus (i.e., $d\delta_n/dn \geq 0$) and $c(0) = 0$, an increasing profit gap between the leader and a follower is sufficient for competition to increase the discounted expected total surplus.

ii) If competition keeps the innovation pass-through on consumer surplus constant (i.e., $d\delta_n/dn = 0$), a decreasing profit gap between the leader and a follower is sufficient for a lessening of competition to increase the expected discounted total surplus.

The intuition of the proposition follows from observing that an increase in the number of research labs intensifies innovation competition (see Proposition 2) and thus decreases the value of firms. This makes consumer welfare dominate producer welfare in the total welfare measure, making the relationship between competition and total welfare take the shape of the relationship between competition and consumer welfare. As we shall see in the next section, consumer and total welfare might align even in the absence of research labs (i.e., $m = 0$), but this is not generally true. The fact that the relationships between competition and consumer and total welfare do not generally align has implications for competition policy, as different criteria for the evaluation of horizontal mergers (i.e., based on consumer welfare or total welfare) may lead to different policy recommendations.

4 An Illustrative Example

In this section, we parameterize the model and simulate the impact of changes in the number of large firms on market outcomes. The purpose of this exercise is to illustrate our results by providing examples that show, first, that the relationship between market structure and the pace of innovation is complex; and second, that a lessening of competition can enhance consumer surplus despite short-run price effects. We consider the case without labs, $m = 0$, unless otherwise noted. Henceforth, we drop the m subscript for ease of notation.

4.1 Parameters

We consider a market for a homogeneous good, where firms compete in quantity (Cournot competition), and market demand is given by $Q = a/P$, with $a > 0$ and $P \leq \bar{P}$. Firms also compete developing a sequence of cost-saving innovations. Each innovation provides the innovating firm with a marginal cost advantage, reducing the leader's marginal cost by a factor of $\beta \in (0, 1)$. The R&D cost function is given by $c(x_i) = \gamma_0 + \gamma_1^{-1} x_i^{\gamma_1}$, where $\gamma_0 \geq 0$ represents the fixed costs of performing R&D and $\gamma_1 > 1$.

We denote, at any instant of time, the marginal cost of the followers by mc and the marginal cost of the leader by $\beta \cdot mc$. The equilibrium market price is $p_n = mc(\beta + n)/n$, which depends on the follower's marginal cost of production, the size of the leader's cost advantage, and the number of followers in the market. As expected, the equilibrium market price is decreasing in n and increasing in both β and mc. Similarly, equilibrium profits are given by

$$\pi_n^l = a\frac{(n(1-\beta)+\beta)^2}{(\beta+n)^2}, \qquad \pi_n^f = a\frac{\beta^2}{(\beta+n)^2},$$

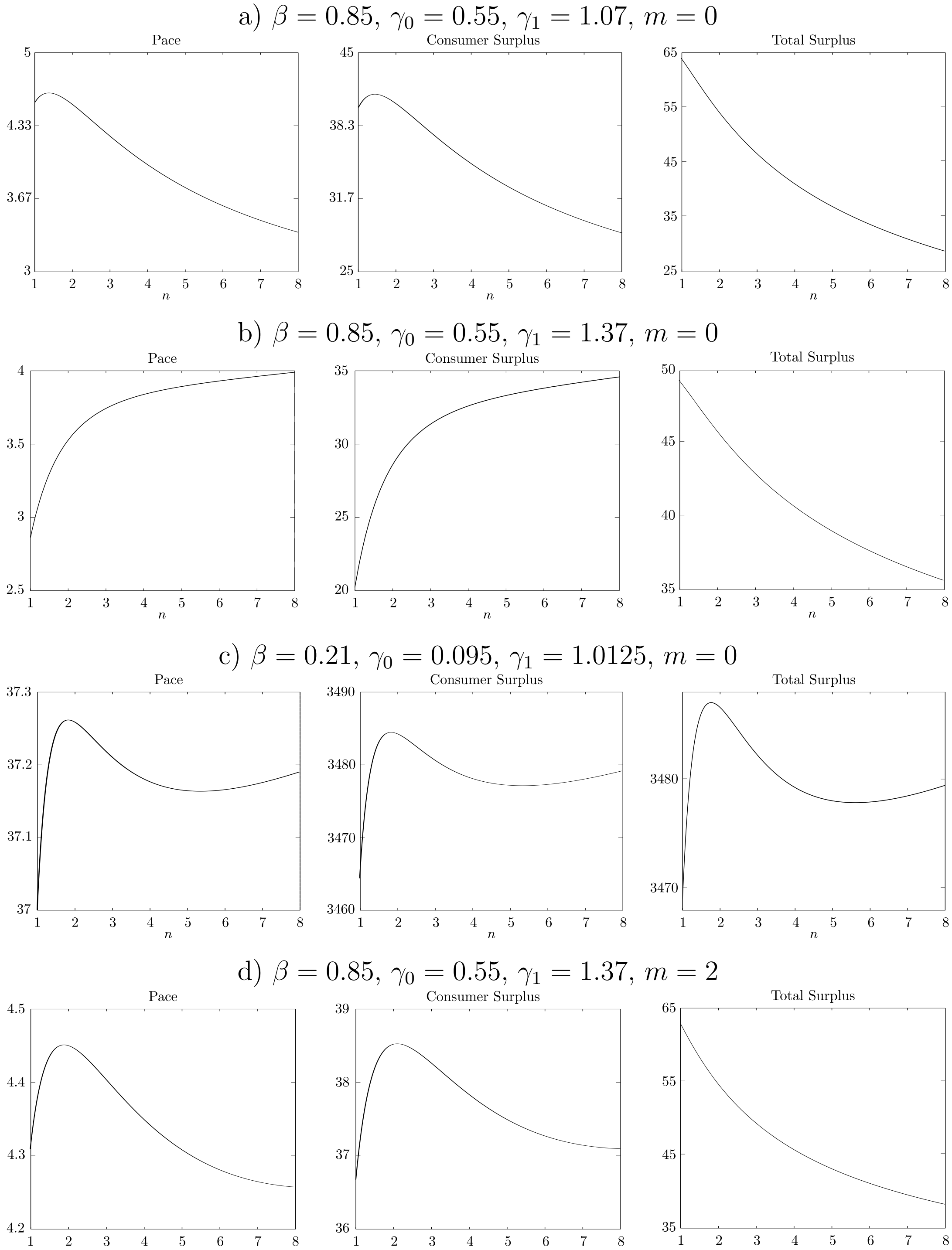
which do not depend on the current marginal cost, nor the number of innovations that have taken place (i.e., payoffs are stationary in the number of innovations). Profits do depend, however, on the number of large firms and the size of the leader's cost advantage, β. These equilibrium profits imply that the profit gap is positive, $\Delta\pi_n \equiv \pi_n^l - \pi_n^f > 0$; decreasing in the number of followers (n), $d\Delta\pi_n/dn < 0$; and increasing in the cost advantage of the leader, $d\Delta\pi_n/d\beta < 0$. As discussed above, a decreasing profit gap may lead to scenarios in which consumer surplus is decreasing in the number of large firms (see Proposition 5).

Finally, to capture the role of the pace of innovation on the path of prices faced by consumers, we make use of the expected discounted consumer surplus defined in equation (5). The flow of consumer surplus when the market price is p_n is given by $cs_n = a \log \bar{P} - a \log p_n$, and an innovation increases the flow of consumer surplus by $\delta \equiv -a \log \beta > 0$.

4.2 Results

Using this setup, we provide four numerical examples to illustrate our results. In Table 3.a (see Figure 2.a) we show market outcomes for a set of parameters

Figure 2: Market-outcome comparison for different numbers of followers and parameter values



Notes: Fixed parameter values are $r = 0.03$, $a = 60$, and $mc = 10$. n is the number of large firm followers, Pace is the pace of innovation, Consumer Surplus is rCS_n (the flow of the expected discounted consumer surplus), Total Surplus is rTS_n (the flow of the expected discounted total surplus, i.e., $rV_n + nrW_n + mrL_n + rCS_n$).

Table 3: Market-outcome comparison for different numbers of followers and parameter values

a) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.07$, $m = 0$

n	λ_n	rCS_n	rV_n	rW_n	rTS_n
1	4.543	39.050	12.480	12.447	63.977
2	4.527	39.363	4.976	4.944	54.226
3	4.237	36.725	2.500	2.469	46.632
4	3.977	34.285	1.388	1.358	41.105
5	3.768	32.312	0.794	0.765	36.932
6	3.602	30.738	0.441	0.412	33.649
7	3.468	29.470	0.213	0.184	30.973
8	3.360	28.435	0.058	0.029	28.728

b) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.37$, $m = 0$

n	λ_n	rCS_n	rV_n	rW_n	rTS_n
1	2.865	22.682	13.302	13.258	49.243
2	3.531	29.645	5.412	5.375	45.808
3	3.747	31.939	2.773	2.741	42.935
4	3.842	32.971	1.578	1.548	40.743
5	3.896	33.562	0.936	0.909	39.041
6	3.934	33.977	0.551	0.525	37.680
7	3.965	34.314	0.302	0.277	36.558
8	3.994	34.617	0.131	0.108	35.609

c) $\beta = 0.21$, $\gamma_0 = 0.095$, $\gamma_1 = 1.0125$, $m = 0$

n	λ_n	rCS_n	rV_n	rW_n	rTS_n
1	37.048	3464.601	2.222	2.191	3469.014
2	37.258	3484.460	0.716	0.685	3486.547
3	37.219	3480.852	0.351	0.320	3482.163
4	37.191	3478.311	0.203	0.172	3479.204
5	37.181	3477.375	0.127	0.097	3477.986
6	37.182	3477.487	0.083	0.052	3477.881
7	37.190	3478.233	0.054	0.023	3478.447
8	37.202	3479.359	0.033	0.003	3479.415

d) $\beta = 0.85$, $\gamma_0 = 0.55$, $\gamma_1 = 1.37$, $m = 2$

n	λ_n	rCS_n	rV_n	rW_n	rTS_n
1	4.315	36.824	12.595	12.561	62.868
2	4.456	38.670	5.131	5.100	54.628
3	4.410	38.412	2.631	2.602	49.304
4	4.356	37.984	1.494	1.467	45.694
5	4.315	37.645	0.881	0.856	43.084
6	4.287	37.421	0.513	0.489	41.096
7	4.271	37.295	0.274	0.251	39.518
8	4.263	37.246	0.109	0.087	38.224

Notes: Fixed parameter values are $r = 0.03$, $a = 60$, and $mc = 10$. n is the number of large firm followers, λ_n is the pace of innovation, CS_n is the expected discounted consumer surplus, V_n is the value of being the leader, W_n is the value of being a follower, and TS_n is the expected discounted total surplus (i.e., $V_n + nW_n + mL_n + CS_n$).

that create an inverted-U relationship between the pace of innovation and the number of large firm followers (n). A similar inverted-U relationship is found for the expected discounted consumer surplus. This example shows that a lessening of competition may enhance consumer surplus by increasing the pace of innovation—for instance, when going from $n = 3$ to $n = 2$ large firm followers—even though market concentration increases prices in the short run. The gains in consumer surplus arise from consumers enjoying more frequent price reductions—caused by a greater pace of innovation—that more than compensate for the short-run price effects.

Result 1. A lessening of competition may enhance consumer surplus even if it increases prices in the short run.

In Tables 3.b and 3.c (see Figures 2.b and 2.c, respectively), we show examples in which the pace of innovation varies monotonically (Table 3.b) or non-monotonically (N-shaped in Table 3.c) with respect to the number of followers. These examples illustrate the complex relationship that exists between the number of large firms and the pace of innovation. As discussed in Section 3, the shape of this relationship is given by the relative importance of two separate effects created by a change in competition. On the one hand, a lessening of competition may increase the profit gap between the leader and followers—increasing the incentives to innovate; on the other hand, it reduces the number of firms performing R&D.

Result 2. The relationship between the pace of innovation and the number of firms can be monotonic or non-monotonic (e.g., inverted-U or N shaped).

In Proposition 4, we argue that a decreasing profit gap becomes sufficient for competition to decrease the pace of innovation when the number of research labs, m, is sufficiently large. In these examples, however, we find that the number of research labs needed to generate a decreasing relationship between competition and the pace of innovation can be as small as zero (e.g., Table 3.a and Table 3.c). In some cases, however, we do find that a profit gap that decreases in the number of large firms is insufficient for competition to decrease the pace of innovation (e.g., Table 3.b). To illustrate how the presence of research labs can impact market outcomes, Table 3.d (see Figure 2.d) uses the same parameters as Table 3.b, but adds two research labs to the analysis ($m = 2$). The table shows that it may only take a small number of research labs ($m = 2$ in this case) to transform the relationship between the number of large firms and the pace of innovation.

Result 3. A profit gap that decreases in the number of large firms in conjunction with a small number of labs may generate a decreasing relationship between competition and innovation.

Lastly, the example in Table 3.b shows that competition may increase consumer welfare while decrease total welfare. That is, the relationships between competition and consumer and producer surplus need not be aligned. This finding has implications for competition policy, as the conclusions of an antitrust authority evaluating a merger may depend on whether it uses a consumer welfare or total welfare criterion. Proposition 6 shows that the discrepancy between the relationships between competition and consumer and total welfare disappears when the number of research labs is sufficiently large. As in the previous paragraph, we can use the example in Table 3.d to see this result at work.

Result 4. The relationships between competition and consumer and producer surplus need not be aligned. A small number of research labs may be sufficient for them to become aligned.

5 Concluding Remarks

We studied the impact of competition on market outcomes in innovative industries. A lessening of competition affects R&D outcomes both directly by reducing the number of firms performing R&D and indirectly by changing the product market profits. The relationship among these effects is complex and may lead to scenarios in which a lessening of competition increases an industry's pace of innovation and consumer surplus in the long run.

Although the relationship between competition and innovation may take various shapes, the product market game being played by firms puts shape restrictions on this relationship. We provided conditions for when competition increases or decreases the pace of innovation as well as consumer welfare. These conditions are based on product market payoffs, and highlight the importance of the product market for analyzing the impact of competition on R&D outcomes.

Our results have three broader implications. First, the results are constructive in that they isolate a specific property of the product market payoffs that is key for understanding the relationship between innovation and competition. Second, the results show how product market payoffs restrict the relationship between competition and innovation, calling for flexible demand systems in model-based empirical studies. That is, product market games that ex-ante restrict the relationship between competition and innovation should be avoided by researchers conducting empirical work. Lastly, while this is not a paper about mergers in innovative industries—e.g., we do not explicitly model the asymmetries caused by mergers—these results show the various mechanisms by which a change in competition caused by a merger would impact market outcomes.

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Appendix

A Value Functions

Let $V_{n,m}$ represent the value of being the market leader, $W_{n,m}$ the value of being a follower, and $L_{n,m}$ the value of being a research lab when there are n followers and m labs in the industry. At time t, we can write the payoffs of the different types of firms as follows:

$$
\begin{aligned}
V_{n,m} &= \int_t^\infty (\pi_n^l + \lambda_{n,m} W_{n,m}) e^{-(r+\lambda_{n,m})(s-t)} ds, \\
W_{n,m} &= \max_{x_i} \int_t^\infty (\pi_n^f + x_i V_{n,m} + x_{-i} W_{n,m} - c(x_i)) e^{-(r+\lambda_{n,m})(s-t)} ds, \\
L_{n,m} &= \max_{y_i} \int_t^\infty (y_i(V_{n,m} - W_{n,m} + L_{n,m}) + y_{-i} L_{n,m} - c(y_i)) e^{-(r+\lambda_{n,m})(s-t)} ds,
\end{aligned}
$$

where $\lambda_{n,m} = \sum_i^n x_i + \sum_j^m y_j$ is the industry-wide *pace* or *speed* of innovation, $x_{-i} = \lambda_{n,m} - x_i$, and $y_{-i} = \lambda_{n,m} - y_i$. To understand the firms' payoffs, fix any instant of time $s > t$. With probability $\exp(-\lambda_{n,m}(s-t))$, no innovation has arrived between t and s. At that instant, the leader receives the flow payoff π_n^l and the expected value of becoming a follower, $\lambda_{n,m} W_{n,m}$. Each follower receives the flow payoff π_n^f; innovates at rate x_i; earns an expected payoff of $x_i V_{n,m}$; pays the flow cost of its R&D, $c(x_i)$; and faces innovation by other firms at rate x_{-i}. Note that since all large firms are symmetric, they value an innovation in $V_{n,m} - W_{n,m}$. These valuations, in conjunction with the auction format, imply that labs sell their innovations at price $V_{n,m} - W_{n,m}$ in equilibrium. Labs obtain this revenue at rate y_i; pay the flow cost of their R&D, $c(y_i)$; and face innovation by other firms at rate y_{-i}. All of these payoffs are discounted by $\exp(-r(s-t))$.

B Preliminary Results

Lemma 1. *The function $f(z)$ implicitly defined by $c'(f(z)) = z$ satisfies:*

1. *$f(z) > 0$ for all $z > 0$ and $f(0) = 0$.*
2. *$f'(z) > 0$ for all $z \geq 0$. Also, if $c'''(x) \geq 0$, $f''(z) \leq 0$, i.e. f is concave.*
3. *Let $h(z) = (n+1)zf(z) - c(f(z))$ for $z \geq 0$. Then $h'(z) = (n+1)f(z) + nzf'(z) > 0$ for all $z \geq 0$.*

Proof. 1. $c(x)$ being strictly increasing and differentiable implies $c'(x) > 0$ for all $x > 0$. $c(x)$ being strictly convex implies $c''(x) > 0$ for all $x \geq 0$. Thus, $c'(x)$ is unbounded above and for each z there exists a unique value of $x = f(z) > 0$ such that $c'(x) = z$. Moreover, because $c'(0) = 0$, then $f(0) = 0$.
2. The first result follows from the derivative of the inverse function being equal to $f'(z) = 1/c''(f(z))$ in conjunction with the strict convexity of $c(x)$. The second from $f''(z) = -c'''(f(z))/(c''(f(z))^3)$ and the assumption $c'''(x) \geq 0$.

3. Differentiating h and using $c'(f(z)) = z$ delivers $h'(z) = (n+1)f(z) + nzf'(z)$, which is positive by claims 1 and 2. $\square$

Lemma 2. *The discounted expected consumer surplus is given by equation (5).*

Proof. Consider an asset that pays the consumer surplus flow at every instant of time. Starting from a consumer surplus cs_n, the value of this asset is given by

$$rA(cs_n) = cs_n + \lambda_{n,m}(A(cs_n') - A(cs_n)) \tag{7}$$

where cs_n' is the consumer surplus after an innovation arrives. Using the condition that $cs_n' = cs_n + \delta_n$, we guess and verify that equation (5) solves equation (7), i.e., $A(cs_n) = CS_n$, proving the result. $\square$

C Proofs

Proof of Proposition 1. Using the first order condition (see equation (4)), we find that the equilibrium values for the leader and followers are given by

$$\begin{aligned} rV_{n,m} &= \pi_n^l - (n+m)(V_{n,m} - W_{n,m})f(V_{n,m} - W_{n,m}) \\ rW_{n,m} &= \pi_n^f + (V_{n,m} - W_{n,m})f(V_{n,m} - W_{n,m}) - c(f(V_{n,m} - W_{n,m})). \end{aligned}$$

Subtracting these equations and defining $Z_{n,m} \equiv V_{n,m} - W_{n,m}$ we obtain

$$rZ_{n,m} = \Delta\pi_n - (n+m+1)Z_{n,m}f(Z_{n,m}) + c(f(Z_{n,m})). \tag{8}$$

To prove existence and uniqueness of an equilibrium with $Z_{n,m} > 0$, note that the left-hand side of equation (8) is strictly increasing in $Z_{n,m}$ and ranges from 0 to ∞. Lemma 1.1 implies that the right-hand side of equation (8) is strictly decreasing in $Z_{n,m}$, taking the value of $\Delta\pi_n + c(0) > 0$ when $Z_{n,m} = 0$. Thus, the two functions intersect once at a positive value of $Z_{n,m}$, proving the result. $\blacksquare$

Proof of Proposition 2. Using implicit differentiation in equation (8), we reach the following results:

i) The derivative of $Z_{n,m}$ with respect to $\Delta\pi_n$ is given by

$$\frac{dZ_{n,m}}{d\Delta\pi_n} = \frac{1}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} > 0.$$

Since $x^*_{n,m} = f(Z_{n,m})$ and $\lambda_{n,m} = (n+m)f(Z_{n,m})$, Lemma 1.2 implies that both are increasing in $\Delta\pi_n$.

ii) The derivative of $Z_{n,m}$ with respect to m is given by

$$\frac{dZ_{n,m}}{dm} = \frac{-Z_{n,m}f(Z_{n,m})}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} < 0.$$

Thus, an increase in m decreases a firm's R&D investment. The derivative of the pace of innovation with respect to m is

$$\begin{aligned}\frac{d\lambda_{n,m}}{dm} &= f(Z_{n,m}) + (n+m)f'(Z_{n,m})\frac{dZ_{n,m}}{dm} \\ &= \frac{rf(Z_{n,m}) + (n+m+1)f(Z_{n,m})^2}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})} > 0.\end{aligned}$$

proving that the pace of innovation increases with m. ■

Proof of Proposition 3. Using implicit differentiation in equation (8) we obtain $dZ_{n,m}/dn$. By replacing it in

$$\frac{d\lambda_{n,m}}{dn} = f(Z_{n,m}) + (n+m)f'(Z_{n,m})\frac{dZ_{n,m}}{dn}, \tag{9}$$

we find

$$\frac{d\lambda_{n,m}}{dn} = \frac{rf(Z_{n,m}) + (n+m+1)f(Z_{n,m})^2 + (n+m)f'(Z_{n,m})\frac{d\Delta\pi_n}{dn}}{r + (n+m+1)f(Z_{n,m}) + (n+m)Z_{n,m}f'(Z_{n,m})}. \tag{10}$$

If $\Delta\pi_n$ satisfies $d\Delta\pi_n/dn > 0$ (i.e., if $\Delta\pi_n$ has an increasing profit gap), then the derivative is positive. Hence, a reduction in the number of large firms leads to a reduction in the pace of innovation. ■

Proof of Proposition 4. A necessary condition for equation (10) to be negative is $d\Delta\pi_n/dn < 0$. For sufficiency, we need to show that there exists an $\bar{m}$ such that $m > \bar{m}$ implies $d\lambda_{n,m}/dn < 0$. Since the denominator of (10) is positive, $d\lambda_{n,m}/dn < 0$ is equivalent to

$$\frac{r}{n+m}\frac{f(Z_{n,m})}{f'(Z_{n,m})} + \frac{n+m+1}{n+m}\frac{f(Z_{n,m})^2}{f'(Z_{n,m})} < -\frac{d\Delta\pi_n}{dn}.$$

$d\Delta\pi_n/dn < 0$ guarantees that right-hand side of the inequality is always positive. Given that $f(0) = 0$ and $f'(0) > 0$ (see Lemma 1), and $dZ_{n,m}/dm < 0$, it is sufficient to show that $\lim_{m\to\infty} Z_{n,m} = 0$ for the inequality to hold.

For any small $\epsilon > 0$, pick $Z_\epsilon \in (0, \epsilon)$. By Proposition 1, equation (8) has a unique solution. Using (8), define m_ϵ to be

$$m_\epsilon = \frac{\Delta\pi_n + c(f(Z_\epsilon)) - (r + (n+1)f(Z_\epsilon))Z_\epsilon}{f(Z_\epsilon)Z_\epsilon},$$

which is always well defined (but possibly negative). Thus, take any decreasing sequence of Z_ϵ converging to zero. For each element of the sequence, there exists an increasing sequence m_ϵ that delivers Z_ϵ as an equilibrium. Thus, $\lim_{m\to\infty} Z_{n,m} = 0$ and the result follows. ■

Proof of Proposition 5. i) See text.

ii) Using the definition $\lambda_{n,m} = (n+m)x^*_{n,m}$ and the assumption that $d\delta_n/dn = 0$, we re-write $\frac{dCS_n}{dn} < 0$ as:

$$\frac{dcs_n}{dn} < -\frac{\delta_n}{r}\left((n+m)\frac{dx^*_{n,m}}{dn} + x^*_{n,m}\right).$$

We show that when m is sufficiently large, a profit gap that is decreasing in the number of firms is sufficient to guarantee that the parenthesis in the expression above goes to $-\infty$, which ensures that the condition holds, as dcs_n/d_n is finite. From Proposition 2, we know that $x^*_{n,m}$ decreases with m. Now, observe

$$(n+m)\frac{dx^*_{n,m}}{dn} = \frac{\frac{d\Delta\pi_n}{dn} - Z_{n,m}f(Z_{n,m})}{\frac{r}{n+m} + \frac{n+m+1}{n+m}f(Z_{n,m}) + Z_{n,m}f'(Z_{n,m})}.$$

From the proof of Proposition 4 we know that $\lim_{m\to\infty} Z_{n,m} = 0$. From Lemma 1, we also know that that $f(0) = 0$ and $f'(0) > 0$. Therefore, when the profit gap is decreasing in the number of firms (i.e., $d\Delta\pi_n/dn < 0$) we have $\lim_{m\to\infty}(n+m)\frac{dx^*_{n,m}}{dn} = -\infty$ and the result follows. ■

Proof of Proposition 6. Differentiate the total surplus, rTS_n, with respect to the number of large firm followers n to obtain:

$$\frac{\partial(rTS_n)}{\partial n} = \frac{\partial ts_n}{\partial n} + \frac{\delta_n}{r}\frac{\partial\lambda_{n,m}}{\partial n} + \frac{\lambda_{n,m}}{r}\frac{\partial\delta_n}{\partial n} - c\left(x^*_{n,m}\right) - (n+m)\,c'\left(x^*_{n,m}\right)\frac{\partial x^*_{n,m}}{\partial n}.$$

Observe that the term $\partial ts_n/\partial n$ is positive by Assumption 2. To prove statement i), observe that convexity of the R&D cost function implies $c\,(x) < c\,(0) + xc'\,(x)$. Using convexity, the assumption $c(0) = 0$, $\partial\lambda_{n,m}/\partial n = x^*_{n,m} + (n+m)\partial x^*_{n,m}/\partial n$, and the first order condition (4), we can write the following lower bound for $\partial(rTS_n)/\partial n$:

$$\frac{\partial(rTS_n)}{\partial n} > \frac{\partial ts_n}{\partial n} + \frac{1}{r}\frac{\partial\lambda_{n,m}}{\partial n}\left(\delta_n - rZ_{n,m}\right) + \frac{\lambda_{n,m}}{r}\frac{\partial\delta_n}{\partial n}.$$

By assumption of statement i), $\partial\delta_n/\partial n \geq 0$. Thus, for the derivative to be positive we need $\delta_n \geq rZ_{n,m}$. In the proof of Proposition 4, we showed that $\lim_{m\to\infty} Z_{n,m} = 0$. Hence, the derivate is positive for a sufficiently large number of research labs. For statement ii), rearrange the derivative above using the assumptions of the statement to obtain

$$\frac{\partial\left(rTS_n\right)}{\partial n} = \frac{\partial ts_n}{\partial n} + (n+m)\left(\frac{\delta_n}{r} - c'\left(x^*_{n,m}\right)\right)\frac{\partial x^*_{n,m}}{\partial n} + \frac{\delta_n}{r}x^*_{n,m} - c\left(x^*_{n,m}\right).$$

By Proposition 4, a decreasing profit gap with a sufficiently large number of labs m implies $\partial x^*_{n,m}/\partial n < 0$. From the proof of Proposition 5 we know that as $m \to \infty$,

$x^* \to 0$, thus

$$\lim_{m\to\infty} \frac{\partial \left(rTS_n\right)}{\partial n} = \frac{\partial ts_n}{\partial n} - c\left(0\right) + \frac{\delta_n}{r}\left(\lim_{m\to\infty}\left(n+m\right)\frac{\partial x^*_{n,m}}{\partial n}\right),$$

From the proof of Proposition 5 we also know $\lim_{m\to\infty}(n+m)\frac{dx^*_{n,m}}{dn} = -\infty$. Thus, by continuity in m, for a sufficiently large number of labs the derivative is negative and the result follows. ■

Online Appendix: Not For Publication

Competition and Innovation: The Role of the Product Market

Guillermo Marshall and Álvaro Parra

Leader Innovation

Our baseline model abstracted away from the possibility that the leader invests in R&D by assuming that old patents were not enforceable—enabling followers to imitate them—and thus keeping the leader only one step ahead of all followers. This extension shows that the profit gap remains important when market leaders can invest in R&D to increase their technological lead. In particular, a weakly increasing profit gap is still sufficient for competition to increase the pace of innovation, and a decreasing profit gap is still necessary but not sufficient for competition to lead to lower levels of R&D

Following Acemoglu and Akcigit (2012), we modify the baseline model by assuming that followers make radical innovations, making the replaced leader's product obsolete and available to unsuccessful followers; and, that market leaders invest in R&D to increase the quality of their product, which increases their profit flow. In concrete terms, we assume that the leader may be k steps ahead of the followers, receiving a profit flow of π_n^k. We assume $\pi_n^{k+1} > \pi_n^k$, so that a larger technological gap leads to a higher profit flow. As before, each follower innovates at a rate x_n^f at a flow cost of $c(x_n^f)$. Similarly, the leader can now achieve an innovation at a rate x_f^l at a flow cost $c(x_n^l)$. For this extension, we also assume $c'''(x) \geq 0$.

Although our results will apply to environments in which the leader may improve the quality of its product multiple times, for illustration purposes, we examine a situation in which the leader can increase the quality of its product only once (i.e., $k \in \{1, 2\}$). In the model, we also assume that the followers' profit flow remains constant independently of how many steps ahead the leader is. Then, the followers value function is still represented by equation (2). Let V_n^k be the value of being a leader that has innovated $k \in \{1, 2\}$ times. The leader's value equations are represented by

$$rV_n^1 = \max_{x_n^l} \pi_n^1 + x_n^l \left(V_n^2 - V_n^1\right) - c(x_n^l) + nx_n^f(W_n - V_n^1) \quad (11)$$

$$rV_2 = \pi_n^2 + nx_n^f(W_n - V_n^2), \quad (12)$$

The first equation describes the value of a being a leader that has innovated only once and that is investing in R&D to increase the quality of its product. The second equation describes the value of a leader that has already increased the quality of its innovation, enjoying a profit flow π_n^2. Note that because we assume it is infeasible for the leader to increase the product quality a second time and because developing a radical innovation replaces the current technology that the leader possess, the leader chooses not to invest in R&D when it is two steps ahead (replacement effect).

The first order condition for the followers is given by equation (4), whereas the first order condition for the leader that is one step ahead is given by

$$c_x\left(\hat{x}_n^l\right) = V_n^2 - V_n^1. \tag{13}$$

Similar to the followers in the baseline model, the leader will invest in R&D when the marginal cost of R&D equals the incremental rent of achieving an innovation, $V_n^2 - V_n^1$.

Define $\Delta_n^f = \pi_n^1 - \pi_n^f$ and $\Delta_n^l = \pi_n^2 - \pi_n^1$ to be the profit gap that exists between a one-step ahead leader and its followers, and the profit gap that exists between being a two-step ahead leader and a one-step ahead leader. Let $\lambda_n^2 = nx_n^f$ and $\lambda_n^1 = nx_n^f + x_n^l$ be the pace of innovation when the leader is two and one step ahead, respectively. We start by showing that the profit gap has a similar role to that in the baseline model.

Proposition 7 (Innovating leader)**.** *There exists a unique symmetric equilibrium, which is characterized by the solution of equations (2), (4), (11), (12), and (13). An increase in the profit gap of the leader Δ_n^l increases R&D investments of the leader and followers; consequently, it increases the pace of innovation in the economy. An increase in the profit gap of the followers Δ_n^f increases the followers' R&D, but decreases the R&D of the leader. The pace of innovation, however, increases with Δ_n^f regardless of whether the leader is one or two-steps ahead.*

An increase of the profit gap of any firm that is ahead in the quality ladder increases the reward to innovate for all the firms that lag behind. This increase in reward, thus, increases the R&D incentives of every firm aiming to reach that state. For instance, an increase in the profit gap of a one-step ahead leader increases not only its R&D incentives but also the incentives of followers aiming to become a one-step ahead leader.

In contrast, an increase in the profit gap of firms that are behind in the quality ladder does not lead to higher rewards for innovation for the firm ahead. On the contrary, the increase in profit gap of laggard firms induces them to perform more R&D, increasing the competition of the firm ahead. In turn, the increased competition faced by the firm ahead, decreases its incremental rent and incentives to perform R&D. This countervailing effect is, however, of second order as the pace of innovation increases with a larger profit gap of the followers.

Proposition 8 (Innovating leader II)**.** *Profit gaps Δ_n^f and Δ_n^l that are weakly increasing in n are sufficient to guarantee that market concentration leads to a slower pace of innovation. Similarly, decreasing profits gaps are necessary but not sufficient for market concentration to lead to higher innovation pace.*

Although this formulation abstracts away from research labs, it is not hard to see that the sufficiency result presented in Proposition 4 can be extended to this framework. Research labs mimic the incentives of the followers, magnifying their response in R&D investments due to changes in market concentration. Because a profit gap Δ_n^f that is decreasing in n tends to increase the followers' R&D when the product market concentrates, competition can lead to decreased R&D outcomes when there is a sufficiently large number of research labs and there are decreasing profit gaps.

Omitted Proofs

Proof of Proposition 7. Define the incremental rent of the leader to be $H_n = V_n^2 - V_n^1$ and the incremental rent of followers $Z_n = V_n^1 - W_n$. Using the inversion defined in Lemma 1 we write $\hat{x}_n^l = f(H_n)$ and $\hat{x}_n^f = f(Z_n)$. Subtracting (11) from (12) delivers:

$$rH_n = \Delta_n^l - f\left(H_n\right)H_n + c\left(f\left(H_n\right)\right) - nf\left(Z_n\right)H_n.$$

Similarly, subtracting (11) and (2) delivers:

$$rZ_n = \Delta_n^f + f\left(H_n\right)H_n - c\left(f\left(H_n\right)\right) - (n+1)\,f\left(Z_n\right)Z_n + c\left(f\left(Z_n\right)\right)$$

We need to show that there exists unique positive values of H_n and Z_n that simultaneously solve the equations above. Rewrite the first equation as:

$$f\left(Z_n\right) = \frac{\Delta_n^l - \left(f\left(H_n\right) + r\right)H_n + c\left(f\left(H_n\right)\right)}{nH_n}$$

Using Lemma 1 we can show that this expression defines a negative, monotonic and continuous relation between Z_n and H_n. In particular, observe that if $H_n \to 0$, then $Z_n \to \infty$. Also, if $H_n \to \infty$, then $Z_n < 0$. Rewrite the expression for rZ_n as:

$$rZ_n + (n+1)\,f\left(Z_n\right)Z_n - c\left(f\left(Z_n\right)\right) = \Delta_n^f + f\left(H_n\right)H_n - c\left(f\left(H_n\right)\right)$$

Lemma 1 implies a increasing, monotonic and continuous relation between Z_n and H_n. Observe that $H_n = 0$ implies $Z_n > 0$. Also, $H_n \to \infty$ implies $Z_n \to \infty$. Therefore, the relation described by both equations must intercept and, because both expressions are monotonic, there is a unique intersection. Thus, an equilibrium exists and is unique.

To study the relation between the profit gaps and firms investments and pace of innovation we need to understand the impact of the gaps in the incremental rent, i.e., $\frac{dH_n}{d\Delta_n^k}$ and $\frac{dZ_n}{d\Delta_n^k}$ for $k\{l,f\}$. For this we make use of the implicit function theorem. Define $g : \mathbb{R}^2 \to \mathbb{R}^2$ where

$$g_1(H_n, Z_n) = \Delta_n^l - (f(H_n) + r)H_n + c(f(H_n)) - nf(Z_n)H_n$$
$$g_2(H_n, Z_n) = \Delta_n^f + f(H_n)H_n - c(f(H_n)) - ((n+1)f(Z_n) + r)Z_n + c(f(Z_n)).$$

Then, an equilibrium is defined by $g(H_n, Z_n) = 0$ and the implicit function theorem implies (in matrix notation):

$$\left[\frac{dH_n}{d\Delta_n^f}, \frac{dH_n}{d\Delta_n^l}; \frac{dZ_n}{d\Delta_n^f}, \frac{dZ_n}{d\Delta_n^l}\right] = -\left(A^{-1}\right)B \tag{14}$$

where

$$A = \begin{bmatrix} \frac{\partial g_1}{\partial H_n} & \frac{\partial g_1}{\partial Z_n} \\ \frac{\partial g_2}{\partial H_n} & \frac{\partial g_2}{\partial Z_n} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{\partial g_1}{\partial \Delta_n^f} & \frac{\partial g_1}{\partial \Delta_n^l} \\ \frac{\partial g_2}{\partial \Delta_n^f} & \frac{\partial g_2}{\partial \Delta_n^l} \end{bmatrix}. \tag{15}$$

Using Lemma 1, we find that

$$A = -\begin{bmatrix} r + nf(Z_n) + f(H_n) & nf'(Z_n)H_n \\ -f(H_n) & r + (n+1)f(Z_n) + nf'(Z_n)Z_n \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.$$

The inverse of A is given by

$$A^{-1} = -\frac{1}{|A|}\begin{bmatrix} r + (n+1)f(Z_n) + nf'(Z_n)Z_n & -nf'(Z_n)H_n \\ f(H_n) & r + nf(Z_n) + f(H_n) \end{bmatrix}$$

where $|A|$ is equal to

$$(r + nf(Z_n) + f(H_n))(r + (n+1)f(Z_n) + nf'(Z_n)Z_n) + nf'(Z_n)f(H_n)H_n,$$

which is positive. Then, using equation (14), we compute the derivatives:

$$\begin{bmatrix} \frac{dH_n}{d\Delta_n^f} & \frac{dH_n}{d\Delta_n^l} \\ \frac{dZ_n}{d\Delta_n^f} & \frac{dZ_n}{d\Delta_n^l} \end{bmatrix} = \frac{1}{|A|}\begin{bmatrix} -nf'(Z_n)H_n & r + (n+1)f(Z_n) + nf'(Z_n)Z_n \\ r + nf(Z_n) + f(H_n) & f(H_n) \end{bmatrix},$$

proving the statements with respect to firms' R&D investments and that an increase of Δ_n^l leads to a higher innovation pace. To show the relation between the profit gap of the followers and λ_n^2 observe

$$\begin{aligned} \frac{d\lambda_n^2}{d\Delta_n^f} &= nf'(Z_n)\frac{dZ_n}{d\Delta_n^f} + f'(H_n)\frac{dH_n}{d\Delta_n^f} \\ &= nf'(Z_n)\frac{r + nf(Z_n) + f(H_n) - f'(H_n)H_n}{|A|}. \end{aligned}$$

By Lemma 1 the function $f(z)$ is concave and $f(0) = 0$. Together they imply $f(z) \geq f'(z)z$; thus, the derivative is positive, and the result follows. ■

Proof of Proposition 8. As in the previous proof, we make use of the implicit function theorem. Let $g(H_n, Z_n)$ be the function defined in the proof of Proposition 7. Then, the implicit function theorem implies (in matrix notation)

$$\left[\frac{dH_n}{dn}; \frac{dZ_n}{dn}\right] = -\left(A^{-1}\right) B \tag{16}$$

where A is the matrix defined in (15) and

$$B = \left[\frac{\partial g_1}{\partial n}; \frac{\partial g_2}{\partial n}\right] = \left[\frac{d\Delta_n^l}{dn} - f\left(Z_n\right) H_n; \frac{d\Delta_n^f}{dn} - f\left(Z_n\right) Z_n\right].$$

Using equation (16) we compute the derivatives

$$\frac{dH_n}{dn} = \frac{\psi_{n+1}\left(\frac{d\Delta_n^l}{dn} - f\left(Z_n\right) H_n\right) + nf'\left(Z_n\right)\left(Z_n \frac{d\Delta_n^l}{dn} - H_n \frac{d\Delta_n^f}{dn}\right)}{|A|}$$

$$\frac{dZ_n}{dn} = \frac{f\left(H_n\right)\left(\frac{d\Delta_n^l}{dn} + \frac{d\Delta_n^f}{dn} - f\left(Z_n\right)\left(Z_n + H_n\right)\right) + \psi_n\left(\frac{d\Delta_n^f}{dn} - f\left(Z_n\right) Z_n\right)}{|A|},$$

where $\psi_x = r + xf\left(Z_n\right) > 0$ for all $x > 0$. With these computations we can now prove that the pace of innovation increases in n under increasing profit gaps. Let's start studying the situation in which the leader is two steps ahead, the derivative of λ_n^2 with respect n is given by

$$\begin{aligned}
\frac{d\lambda_n^2}{dn} &= f\left(Z_n\right) + nf'\left(Z_n\right)\frac{dZ_n}{d_n} \\
&= \frac{\left(\psi_n + f\left(H_n\right)\right)\left(f\left(Z_n\right)\psi_{n+1} + nf'\left(Z_n\right)\frac{d\Delta_n^f}{dn}\right) + nf'\left(Z_n\right) f\left(H_n\right)\frac{d\Delta_n^l}{dn}}{|A|}.
\end{aligned}$$

which is positive whenever $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} \geq 0$. Also, we can see that $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} < 0$ are necessary but not sufficient for $\frac{d\lambda_n^2}{dn}$ to be negative.

When the leader performs R&D, i.e., the leader is one step ahead of the followers, the derivative of the pace of innovation is given by:

$$\begin{aligned}
\frac{d\lambda_n^1}{dn} &= \frac{d\lambda_n^2}{dn} + f'\left(H_n\right)\frac{dH_n}{dn} \\
&= \frac{f\left(Z_n\right)\psi_n\psi_{n+1}}{|A|} + \frac{nf'\left(Z_n\right)\left(f\left(H_n\right) + f'\left(H_n\right) Z_n\right) + f'\left(H_n\right)\psi_{n+1}}{|A|}\frac{d\Delta_n^l}{dn} \\
&+ \frac{nf'\left(Z_n\right)\psi_n}{|A|}\frac{d\Delta_n^f}{dn} + \left(f\left(H_n\right) - f'\left(H_n\right) H_n\right)\frac{nf'\left(Z_n\right)\frac{d\Delta_n^f}{dn} + f\left(Z_n\right)\psi_{n+1}}{|A|}.
\end{aligned}$$

By Lemma 1 the function $f(z)$ is concave and $f(0) = 0$; these two conditions imply $f(z) \geq f'(z)z$. Then the derivatives are positive whenever $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} \geq 0$, and $\frac{d\Delta_n^l}{dn}, \frac{d\Delta_n^f}{dn} < 0$ are necessary but not sufficient for $\frac{d\lambda_n^1}{dn}$ to be negative. $\blacksquare$